Asteroseismic analysis of solar-like star KIC 6225718: constraints on stellar parameters and core overshooting

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ABSTRACT

We analyse five seasons of short-cadence data of a solar-type star of spectral type F: KIC 6225718 observed by Kepler. We obtain the power spectrum of this star by applying the Lomb–Scargle periodogram to the smoothed time series. By applying the autocorrelation technique to the power spectrum, we derive the large-frequency separation \(\Delta \nu = 105.78 \pm 0.65 \mu\text{Hz}\) and the frequency of maximum power \(\nu_{\text{max}} = 2301 \pm 21 \mu\text{Hz}\). We identify 33 p modes with angular degrees of \(l = 0\)–2 in the frequency range 1600–2800 \(\mu\text{Hz}\) of the power spectrum with Bayesian Markov Chain Monte Carlo algorithms. In order to determine the parameters of the star accurately, we construct a grid of stellar models with core overshooting using the Yale stellar evolution code and then perform preliminary seismological analysis. With both asteroseismic and non-asteroseismic constraints, the following range of stellar parameters is estimated: mass \(M = 1.10^{+0.04}_{-0.03} M_\odot\), radius \(R = 1.22^{+0.01}_{-0.00} R_\odot\) and age \(t = 3.35^{+0.36}_{-0.75}\) Gyr for this star. In addition, we analyse the effects of overshooting on stellar interiors and find that the upper limit of the overshooting parameter \(\alpha_{\text{ov}}\) is approximately 0.2 for this star.

Key words: methods: data analysis – stars: fundamental parameters – stars: individual: KIC 625718 – stars: oscillations.

1 INTRODUCTION

Over the past few decades, seismic observations of stars have been performed using ground-based observations (e.g. Fossat et al. 1984; Bertaux et al. 2003; Bouchy et al. 2005; Claudi et al. 2005; Vauclair et al. 2008). However, these data are limited by discontinuities of the observation time and perturbations of the Earth’s atmosphere. In recent years, space missions, such as MOST, CoRoT and Kepler, have provided continuous and high-precision photometric data, which are quite suitable for asteroseismic analysis (Borucki et al. 2007; Matthews 2007; Appourchaux et al. 2008). In particular, solar-like oscillations have been observed for a number of stars (Hekker et al. 2009; Appourchaux et al. 2012) and many of them have been investigated in detail (e.g. Guenther & Demarque 1986; Bi, Basu & Li 2008; Eggenberger et al. 2004; Soriano et al. 2007; Bedding et al. 2010; Gilliland et al. 2010a; Doğan et al. 2013). It has been shown that asteroseismology is an excellent tool for probing the internal structures of stars, testing physical processes in stellar interiors and determining stellar parameters of pulsators (Bedding et al. 2011; Deheuvels et al. 2012; Silva Aguirre et al. 2013).

For F-type stars with mass higher than the Sun and surface temperatures in the range 6000–7600 K, there may be a convective core in the stellar interior. Overshooting from the convective core will feed more H-rich material into the core and thus obviously affect the evolution of stars. Uncertainty in the extension of the convective core due to such overshooting can affect determination of the global parameters of stars by asteroseismology (Di Mauro et al. 2003; Mazumdar et al. 2006). The frequency ratio of small- to large-frequency separations, \(r_{01}\), has been suggested for use to detect the convective core (Ulrich 1986; Roxburgh 1993; Yang & Bi 2007). With this tool, a convective core and core overshooting for ‘Dushera’ (KIC 12009504; \(M = 1.15 \pm 0.04 M_\odot\); Silva Aguirre et al. 2013) and HD 49933 (\(M = 1.28 \pm 0.01 M_\odot\); Liu et al. 2014) have been detected. However, for \(\alpha\) Centauri A (\(M = 1.105 \pm 0.007 M_\odot\)), De Meulenaer et al. (2010) do not favour the existence of a convective core, since a model with a radiative core reproduces the observed \(r_{01}\) significantly better than one with a convective core. In addition, Silva Aguirre et al. (2013) could not clarify whether a convective core exists in ‘Perky’ (KIC 6106415; \(M = 1.11 \pm 0.05 M_\odot\)) or not. In this work, we aim to consider the possibility of measuring the size of the convective core for stars with mass around 1.1 \(M_\odot\) in order to understand whether asteroseismology is the appropriate tool for such a study.
KIC 6225718 (HIP 97527, HD 187637) is an F-type main-sequence star. It had been observed by the Kepler satellite over a long duration from Q0 to Q16.3. Silva Aguirre et al. (2012) extracted the values of $\Delta \nu$ and $v_{\text{max}}$ for KIC 6225718: $\Delta \nu = 105.8 \pm 0.3$ $\mu$Hz and $v_{\text{max}} = 2338 \pm 66$ $\mu$Hz. They also reported mass $M = 1.209^{+0.037}_{-0.034} M_\odot$ and radius $R = 1.256^{+0.014}_{-0.014} R_\odot$ for the star. Meanwhile, Huber et al. (2012) estimated similar values, $\Delta \nu = 105.8 \pm 0.3$ $\mu$Hz and $v_{\text{max}} = 2352 \pm 66$ $\mu$Hz, but slightly different mass and radius, $M = 1.31 \pm 0.11 M_\odot$ and $R = 1.288 \pm 0.038 R_\odot$. Moreover, Chaplin et al. (2014) proposed the values of $\Delta \nu$ and $v_{\text{max}}$ of KIC 6225718 to be $105.8 \pm 0.4$ $\mu$Hz and $2301 \pm 40$ $\mu$Hz. The mass of KIC 6225718 may be greater than those of $\alpha$ Centauri A and ‘Perky’ and close to those of ‘Dushera’ (Silva Aguirre et al. 2013) and HD 49933 (Liu et al. 2014). Thus, there might be a convective core in KIC 6225718 and it might be detectable based on the frequency ratio $r_{\text{D}}$. We note that the individual frequencies $v_{i,j}$ have not been derived by Silva Aguirre et al. (2012), Huber et al. (2012) or Chaplin et al. (2014). In order to carry out asteroseismic analysis in detail and to determine accurate stellar parameters and the size of the convective core for KIC 6225718, it is necessary to extract the individual frequencies of the star.

In Section 2, we present data processing, analysis of observational data and extraction of the oscillation modes of $l = 0$–2. In Section 3, we construct a grid of stellar models with core overshooting and calibrate the models. The asteroseismic diagnostics and the effects of overshooting on stellar models are shown in Section 4. Finally, discussion and conclusions are given in Section 5.

2 OBSERVATIONS AND DATA ANALYSIS

2.1 Atmospheric parameters

Spectroscopic observations provide different metallicities for KIC 6225718, which leads to its recognition as a metal-poor star. The metallicity is $[\text{Fe/H}] = -0.10 \pm 0.12$ dex in Clementini et al. (1999) and $[\text{Fe/H}] = -0.17 \pm 0.06$ dex in Bruntt et al. (2012). Molenda-Żakowicz et al. (2013) used two different spectroscopic methods to analyse the spectra of this star and obtained $[\text{Fe/H}] = -0.23 \pm 0.15$ dex and $[\text{Fe/H}] = -0.07 \pm 0.15$ dex with the ROTFIT and ARES+MOOG codes, respectively.

The luminosity of KIC 6225718 can be estimated by using the following equation (Pijpers 2003):

$$\log \frac{L}{L_\odot} = 4.0 + 0.4 M_{\text{bol}, \odot} - 2.0 \log \pi \ [\text{mas}] - 0.4(V - A_V + BC(V)), \quad (1)$$

where $M_{\text{bol}, \odot} = 4.746$ is the bolometric magnitude of the Sun given by Lejeune, Cuisinier & Buser (1998). The parallax of KIC 6225718 from the Hipparcos satellite (Perryman & ESA 1997) is 18.78 mas (Hindsley et al. 2002). The Johnson $V$ magnitude is also obtained from Hindsley et al. (2002). The variable $A_V$ is the extinction, while $BC(V)$ is the bolometric correction. To obtain the value of $A_V$, the standard relation between extinction and reddening is adopted (Neckel, Klare & Sarcander 1980):

$$A_V = 3.1[(B - V) - (B - V)_0], \quad (2)$$

where the intrinsic colours $(B - V)_0$ and colours $(B - V)$ are obtained based on the statistics of their relations with log $T$ given by Flower (1996). The effective temperature of KIC 6225718 given by Bruntt et al. (2012) is 6250 $\pm$ 70 K. With these data, we estimated that the luminosity of KIC 6225718 is 2.11 $\pm$ 0.11 $L_\odot$. These parameters are summarized in Table 1.

### Table 1. Basic parameters of KIC 6225718.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numerical value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{eff}}$</td>
<td>$6250 \pm 70$ K</td>
<td>Bruntt et al. (2012)</td>
</tr>
<tr>
<td>$L/L_\odot$</td>
<td>$2.11 \pm 0.11$</td>
<td>in this article</td>
</tr>
<tr>
<td>KIC mag</td>
<td>7.50</td>
<td>KASOC website</td>
</tr>
<tr>
<td>Johnson $V$ mag</td>
<td>7.58</td>
<td>Hindsley et al. (2002)</td>
</tr>
<tr>
<td>Parallax</td>
<td>18.78 mas</td>
<td>Hindsley et al. (2002)</td>
</tr>
<tr>
<td>log $g$</td>
<td>$4.32 \pm 0.03$ dex</td>
<td>Bruntt et al. (2012)</td>
</tr>
<tr>
<td>$[\text{Fe/H}]$</td>
<td>$-0.17 \pm 0.06$ dex</td>
<td>Bruntt et al. (2012)</td>
</tr>
</tbody>
</table>
| $-0.10 \pm 0.12$ dex | Clementini et al. (1999)
| $-0.23 \pm 0.15$ dex | Molenda-Żakowicz et al. (2013)
| $-0.07 \pm 0.15$ dex | Molenda-Żakowicz et al. (2013)

2.2 Data preprocessing and the power spectrum

Short-cadence (58.85-s) photometric data for KIC 6225718 are available to the Kepler Asteroseismic Science Consortium (KASC: Kjeldsen et al. 2010) through the Kepler Asteroseismic Science Operations Center (KASOC) data base.\(^1\) The time series data of quarters 6, 7, 8, 9 and 10 have been corrected by the KASC Working Group 1 (WG#1; ‘solar-like oscillating stars’). We correct for the effects of outliers, jumps and drifts based on the short-cadence photometric data corrected by WG#1, following the procedures described by Gilliland et al. (2010b) and García et al. (2011). Median smoothing with one-day width is adopted to process the data corrected by WG#1, shown in Fig. 1. We set the mean value of the observed time series as the referenced value. We subtract the median-smoothed data from the data corrected by WG#1 and subsequently add the referenced mean value to the median-smoothed flux. The smoothed data for the power spectrum calculation are overplotted in Fig. 1. We note that the median smoothing affects frequencies lower than 11.57 $\mu$Hz (1/1 d), but this range would not affect the extraction of the individual frequencies in our study.

We then obtained the power spectrum by applying a Lomb–Scargle periodogram (Lomb 1976; Scargle 1982) to the smoothed data. The raw power spectrum and the power spectrum smoothed with $2 \mu$Hz are shown in Fig. 2. The excess of power around $v_{\text{max}}$ with a comb-like structure is typical of solar-like stars.

2.3 Estimation of the mean large separation and frequency of maximum power

The autocorrelation function (ACF) can be adopted to estimate the mean value of large-frequency separation. The ACF of a discrete time series $G(t_i)$ is

$$A(t_i) = \sum_{k=1}^{N} G(t_i)G(t_{i+k}). \quad (3)$$

According to the Wiener–Khinchin theorem, the ACF of a time series is equal to the Fourier transform of the power spectrum of the time series. As shown by Roxburgh & Vorontsov (2006), Roxburgh (2009) and Mosser & Appourchaux (2009), the peaks in an ACF are located at $T_{\text{max}} = 4kT$, where $k = 1, 2, 3 \ldots$ and $T$ is the sound radius, $T = [c/4\pi]^1$. The large-frequency separation $\Delta \nu_{\text{max}}$ is approximately $1/(2T)$, therefore the peaks in the ACF provide the mean value of $\Delta \nu_{\text{max}}$. Thus we can obtain the function

$$\Delta \nu \approx \frac{1}{2T} = \frac{2}{T_{\text{max}}}. \quad (4)$$

\(^1\) KASOC data base: http://kasoc.phys.au.dk/
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Figure 1. The flux of KIC 6225718. The black line is the time series corrected by WG#1, while the blue line shows the smoothed data in which outliers, jumpers and drifts are corrected. The time is reduced by 55000 MJD.

Figure 2. Power spectrum of KIC 6225718. The black and red lines denote raw PSD and PSD smoothed with 2 \( \mu \text{Hz} \), respectively.

The peaks in the ACF of a power spectrum around the frequency of maximum power \( \nu_{\text{max}} \) can also provide the mean value of the large-frequency separation \( \Delta \nu \) (Barban et al. 2009), which needs much shorter time of numerical computation. The ACF of the power spectrum is adopted to derive the large-frequency separation. After calculating the ACF of the power spectrum, we apply Gaussian smoothing to the ACF with 10 \( \mu \text{Hz} \), which is shown in Fig. 3. The obvious equally spaced peaks in the ACF of the power spectrum provide the mean value of \( \Delta \nu \). As described by Barban et al. (2009), the first peak is located at a frequency equal to half of the large-frequency separation, while the second peak is located at the frequency of the large-frequency separation. Thus, we obtain the large-frequency separation \( \Delta \nu = 105.78 \pm 0.65 \mu \text{Hz} \).

In order to locate the frequency of maximum power, we follow a three-step procedure described by Huber et al. (2009). Fig. 4 presents all three steps. First, we estimate the background of the power spectrum by binning the power spectrum in equal logarithmic bins and smoothing the binned power spectrum with a median filter.

Figure 3. The ACF of the power spectrum obtained by KIC 6225718. The black line is raw ACF, while the red line shows the Gaussian-smoothed ACF with 10 \( \mu \text{Hz} \). The large separation is located at the second peak of the Gaussian-smoothed ACF.
Combining equations (5) and (6), we obtain
\[
\frac{M}{M_\odot} \approx \left( \frac{\Delta \nu}{\Delta \nu_\odot} \right)^{-4} \left( \frac{v_{\text{max}}}{v_{\text{max, } \odot}} \right)^3 \left( \frac{T_{\text{eff}}}{T_{\text{eff, } \odot}} \right)^{3/2}.
\]
(7)

Here, \(v_{\text{max, } \odot} = 3050 \mu \text{Hz}, \Delta \nu_\odot = 134.9 \mu \text{Hz}\) and \(T_{\text{eff, } \odot} = 5777 \text{ K}\).

On the premise of knowing \(\Delta \nu, v_{\text{max}}\) and \(T_{\text{eff}}\) in advance, the range of mass for modelling is estimated to be \(M = 1.02 - 1.20 M_\odot\).

### 2.4 Mode extraction

Several fitting methods to extract multiple frequency modes are discussed by Mathur et al. (2011): Bayesian Markov chain Monte Carlo algorithms (MCMC: Benomar 2008; Handberg & Campante 2011), the maximum-likelihood estimation approach (MLE: Appourchaux, Gizon & Rabello-Soares 1998), the maximum a posteriori approach (MAP: Gaulme, Appourchaux & Boumier 2009), classic pre-whitening of the frequency power spectrum (CLEAN: Bonanno et al. 2008), Period04 based on the Fourier theorem (Lenz & Breger 2004; Jiang et al. 2011) and estimation of the frequencies of the highest peaks in the smoothed power spectrum. The modes frequencies extracted by these methods are in good agreement with each other.

Here we extract the oscillation modes by using the MCMC method and adopt Metropolis–Hasting (MH) based algorithms to estimate the stellar parameters from the power spectrum, following Handberg & Campante (2011). After obtaining the spectra by applying the Lomb–Scargle periodogram method to the corrected time series, as shown in Fig. 2, we smooth the power spectra by 2 \(\mu \text{Hz}\). The oscillation modes can then be extracted from the peaks of the smoothed power spectrum. In solar-like stars, p-mode oscillations are expected to have the approximate relation (Tassoul 1980)
\[

\nu_{n,l} \approx \Delta \nu \left( n + \frac{l}{2} + \epsilon \right) - l(l + 1)D_0,
\]
(8)

where \(D_0\) is related to the interior structure of the star, \(\Delta \nu\) is related to the mean density of the star and the offset \(\epsilon\) is a constant close to \(1/4\) (Mosser et al. 2013). In the asymptotic description, the various large separations do not change with frequency. The frequencies of the p-mode oscillations that obey this description would show vertical, straight ridges in the échelle diagram. The mode frequencies of KIC 6225718 against those frequencies modulo the mean large-frequency separations are plotted in Fig. 5 and the mode frequencies with \(l = 0, 1\) and 2 are regularly arranged in the échelle diagram. The comb-like structure of the power spectrum is an important constraint on the extraction of oscillation modes. The identified modes listed in Table 2 are overplotted in the échelle diagram presented in Fig. 5.
where we adopt values of \( Z_\odot = \) scaleheight at the edge of the core and we estimate the stellar surface abundance ratio \( Z_{1996} \) and heavy-element abundance between about 0.014 and 0.019. In the model calculation, we set the region to be no larger than a fraction \( \alpha \) quantity. Equation (9) is used to limit the extent of the overshoot.

3.1 Input physics

We construct a grid of evolutionary models with core overshooting using the Yale stellar evolution code (YREC7: Demarque et al. 2008) to estimate the parameters of the star. For the microphysics, we use the OPAL opacity table GN93 (Iglesias & Rogers 1996), the low-temperature table AGS05 (Ferguson et al. 2005), the OPAL equation-of-state tables EOS2005 (Rogers & Nayfonov 2002) and the Bahcall nuclear rates (Bahcall, Pinsonneault & Wasserburg 1995). We choose the Eddington grey-atmosphere \( T_\tau \) relation. The models take into account helium and heavy-element diffusion using the coefficient from Thoul, Bahcall & Loeb (1994). The standard mixing-length parameter \( \alpha \) is set to be equal to 1.78. Following the observations, theoretical frequencies of the modes of degree \( l = 0,1,2 \) are calculated for a radial order \( n \) ranging from 13–24 in the frequency range 1600–2800 \( \mu \)Hz for KIC 6225718. The large separation of each model can be calculated based on the formula \( \Delta \nu_{i,l} = \nu_{i,l} - \nu_{i,l-1} \). To restrict the stellar parameters further, we perform a \( \chi^2 \) minimization by a comparison of models with observations (e.g. Eggenberger et al. 2004; Eggenberger, Carrier & Bouchy 2005):

\[
\chi^2 = \sum_{i} \left( \frac{C_{i,\text{theo}} - C_{i,\text{obs}}}{\sigma_{C_i}} \right)^2 ,
\]

where the vector \( C = (L/L_\odot, T_{\text{eff}}, [\text{Fe/H}], \langle \Delta \nu \rangle) \). Here the vector \( C_{\text{theo}} \) presents the theoretical values of these parameters, while \( C_{\text{obs}} \) contains the observational values and \( \sigma \) contains the errors in these observations. The values of the observational quantities with errors are listed in Table 1. The results indicate that the smaller the value of \( \chi^2 \), the higher the probability of fitting the observations.

We then obtain 169 models with \( \chi^2 < 5 \) as candidate models for further pulsation analysis and compare the theoretical frequencies of the candidate models with the extracted frequencies through a \( \chi^2 \) function, which can be defined as

\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\nu_{i,\text{theo}} - \nu_{i,\text{obs}}}{\sigma_{\nu_i}} \right)^2 .
\]

Here the quantity \( \nu \) corresponds to the individual frequencies of modes from observations and modelling and \( \sigma \nu \) denotes the observation errors listed in Table 2.

To compare the results, the better candidate models with \( \chi^2 \leq 30 \) for the overshooting parameters \( \alpha_{\text{ov}} = 0.0, 0.2 \) and 0.6 are listed in Table 4 and Models 31 and 34 with \( \chi^2 > 30 \) are added into the table for comparison. However, for the models with overshooting \( \alpha_{\text{ov}} = 0.4 \), we list the better candidate models with \( \chi^2 \leq 60 \). We can see that the models without overshooting and with overshooting \( \alpha_{\text{ov}} = 0.2 \) have smaller \( \chi^2 \) than those with overshooting \( \alpha_{\text{ov}} = 0.4 \) and 0.6 from Table 4. It is noticed that all the better candidate models have the same mixing-length parameter \( \alpha = 1.90 \), rather than \( \alpha = 1.70 \). Fig. 6 shows the positions of the better candidate models in the H-R diagram.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum value</th>
<th>Maximum value</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M/M_\odot )</td>
<td>1.02</td>
<td>1.20</td>
<td>0.01</td>
</tr>
<tr>
<td>( Z_i )</td>
<td>0.012</td>
<td>0.018</td>
<td>0.001</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.70</td>
<td>1.90</td>
<td>0.2</td>
</tr>
<tr>
<td>( \alpha_{\text{ov}} )</td>
<td>0.0</td>
<td>0.6</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2. Modes identified for KIC 6225718 with 3\( \sigma \) errors. (\( \mu \)Hz)

Table 3. Input parameters for model tracks.
The zero-age main-sequence (ZAMS) models are very similar, independently of overshooting, since they are chemically uniform (Di Mauro et al. 2003). However, when a convective core appears in the stellar interior at the phase of the main sequence, the core overshooting would be sufficient enough to produce chemical mixing in the region where the turbulent motions penetrate from the edge of the convective core to the convectively stable upper layer. The core overshooting enlarges the size of the convective core and transports more hydrogen-rich material into the convective core, which prolongs the lifetime of central hydrogen burning at the phase of the main sequence. Consequently, the models shown in Table 4 with larger overshooting parameters would need a higher age to achieve a similar $\Delta \nu$ or a similar mean density. With the increase of overshooting parameters, the models would achieve larger convective cores characterized by higher central hydrogen abundance ($X_c$) as presented in Table 4.

We list the location of the base of the convective envelope $r_{cc}$ for the better candidate models in Table 4. Due to the existence of a thick convective envelope in F-type stars, the ratio of the extent of overshooting from a convective envelope to the thickness of the convective envelope is small. As discussed in Aerts, Christensen-Dalsgaard & Kurtz (2010), the effect of convective envelope overshoot on stellar evolution is less significant than the effect of core overshoot. Thus, in this work we ignore overshooting from the convective envelope in the models.

### 4 ASTEROSEISMIC DIAGNOSTICS

#### 4.1 The echelle diagrams

In order to investigate the effects of overshooting on oscillation behaviour of stars, we compare the echelle diagrams of models with different overshooting parameters in Fig. 7. The models have the same metallicity $Z_i = 0.015$ but different masses in each group.
Figure 6. Positions of the better candidate models in the H-R diagram. The red, green, blue and black triangles denote the models without and with overshooting $\alpha_{ov} = 0.2, 0.4, 0.6$, respectively.

For models without overshooting (plotted in Fig. 7(a)), there is a slight difference between the models with different mass. Model 6 with a mass of $M = 1.11 \, M_\odot$ matches the observations better than the other two models, in agreement with the result based on $\chi^2$ minimization.

The échelle diagrams of models with overshooting $\alpha_{ov} = 0.2$ shown in Fig. 7(b) are similar to those in Fig. 7(a). The comparison among the models with overshooting $\alpha_{ov} = 0.2$ shows that Model 17 with a mass of $M = 1.11 \, M_\odot$ reproduces the observations slightly better than the other two models.

Compared with models with overshooting $\alpha_{ov} = 0.4$ (plotted in Fig. 7(c)), it is interesting that there is a comparative shift towards the left at low frequencies and that modes almost overlap at high frequencies, especially for the $l = 1$ modes with increasing masses. The departure from observations for models with overshooting $\alpha_{ov} = 0.4$ is much larger than for the models in Fig. 7(a) and (b).

Fig. 7(d) shows the échelle diagrams of models with overshooting $\alpha_{ov} = 0.6$, which match the observations better than those with overshooting $\alpha_{ov} = 0.4$, but are not as good as the models without overshooting or with overshooting $\alpha_{ov} = 0.2$.

4.2 Frequency ratios $r_{01}$

The clear departure of the computed oscillation frequencies from the observed values in Fig. 7 may result from near-surface effects. As discussed by Christensen-Dalsgaard & Thompson (1997), the solar p-mode frequency offset arises from inadequate modelling of the near-surface layers. Kjeldsen, Bedding & Christensen-Dalsgaard (2008) stated that a similar effect must occur in other stars and they provided a method to correct the near-surface effects in stellar models. In this respect, the ratio of small to large separations, $r_{01}$, is an important tool for testing interior processes, which has been demonstrated to cancel out the influence of the outer layers (Oti Floranes, Christensen-Dalsgaard & Thompson 2005; Roxburgh 2005; Silva Aguirre et al. 2011). Due to a large gradient of chemical composition $\nabla_\mu$ and a sudden change of sound speed at the top of the convective core, there will be a corresponding change of $r_{01}$ when the oscillation modes penetrate into the convective core. The relation between the slopes of small separations and the size of the jump in the sound speed at the edge of the convective core is linear (Brandão et al. 2010). Therefore, as they arrive at the convective core, the oscillation modes would achieve a steeper slope of ratio $r_{01}$ than those that propagate in radiative regions. To constrain the models further, we compare the $r_{01}$ values of the models with those of the observations. Here we adopt the five-point smoothed small-frequency separations $d_{01}$ following Roxburgh & Vorontsov (2003):

$$d_{01}(n) = \frac{1}{8}(v_{n-1,0} - 4v_{n-1,1} + 6v_{n,0} - 4v_{n,1} + v_{n+1,0}),$$

and $r_{01}$ can be expressed as

$$r_{01}(n) = \frac{d_{01}(n)}{\Delta v_{n=1}(n)}.$$
r_{\alpha} = 0.0, 0.2, 0.4, 0.6 listed in Table 4.

The $r_{\alpha}$ values of the models listed in Table 4 are calculated. The $r_{\alpha}$ versus mode frequencies as shown in Fig. 8 indicate that the $r_{\alpha}$ of models with overshooting in the range $r_{\alpha} = 0.0–0.2$ match the observation well. Since the $r_{\alpha}$ values of the models with overshooting $r_{\alpha} > 0.2$ are much smaller than the observed values, these models are eliminated. Therefore, the better candidate models with overshooting $r_{\alpha} = 0.0–0.2$ are selected as the best-fitting models, based on which we estimate the parameters for KIC 6225718: $M = 1.10^{+0.04}_{-0.03} M_\odot$, $R = 1.23^{+0.01}_{-0.005} R_\odot$, and $r = 3.35^{+0.36}_{-0.70}$.

From the radii of the convective cores ($r_{c\alpha}$) of models with overshooting $r_{\alpha} = 0.0–0.2$ listed in Table 4, we see that the star has a small or no convective core in the stellar interior. Note that, at approximately 1770 $\mu$Hz, the models without overshooting cannot reproduce the observation well (Fig. 8). Model 14 with overshooting $r_{\alpha} = 0.2$ is the model that best matches the observations. We conclude that the star more likely has a small convective core.

4.3 Comparison between the lower turning points $r_i$ and the radii of convective cores

To investigate further the mode characteristics of models without and with overshooting, we analyse the lower turning points of acoustic waves and compare them with the radii of convective cores $r_{cc}$ (if they exist) of the models. In the Cowling approximation (Cowling 1941), the lower turning points of acoustic waves, $r_i$, can be obtained from the following equation (cf. Unno et al. 1989; Aerts et al. 2010):

$$r_i^2 = c^2(r_{\alpha}) \frac{l(l+1)}{(2\pi v_f^2)}, \quad (15)$$

where $c(r_{\alpha})$ denotes the sound speed at turning points. Takata (2006) proposed that the inner turning point of the high-frequency dipolar $p$ mode of stars with a radiative core is set by the corrected Brunt–Väisälä frequency, which is in contrast to the conventional understanding in the Cowling approximation that the inner turning point of $p$ modes is fixed by the Lamb frequency. As demonstrated in Liu et al. (2014), equation (15) might underestimate the value of the inner turning point $r_i$ due to the approximations in the adiabatic oscillation equations and the large gradient of chemical composition $\nabla_n$ at the top of the core; they introduced a parameter $f_0$ to modify the equation as follows:

$$r_i^2 = f_0^2 c^2(r_{\alpha}) \frac{l(l+1)}{(2\pi v_f^2)}. \quad (16)$$

At a given angular degree, high-frequency oscillation modes penetrate deeper into the star than low-frequency acoustic waves (see Fig. 9) and the coefficient $f_0$ does not change the relative positions between $r_i$ for fixed angular degree modes.

We compare $r_i$ with $r_{c\alpha}$ (if it exists) and analyse the characteristics of the modes for different values of $f_0$. First, we set $f_0 = 1$ in equation (16), which becomes equation (15). For the $l = 1$ modes of the models with overshooting $r_{\alpha} = 0.2$ as shown in Fig. 9(b), the low-frequency modes propagate outside the convective core, while the high-frequency modes could penetrate into the convective core or propagate close to the boundary of the convective core. In this case, the acoustic waves of models with overshooting $r_{\alpha} = 0.2$ will be affected by the sharp feature represented by the boundary of the convective core. The observed ratios $r_{01}$ shown in Fig. 8 do not exhibit an increase at high frequencies as do those of ‘Dushe’ and HD 49933. This indicates that there is no convective core in KIC 6225718, or the core is too small to be detected by asteroseismology in accordance with the theory of Liu et al. (2014).

Under the premise $f_0 = 1$, KIC 6225718 points towards a star without a convective core. In Fig. 9(c) and (d), we compare the radii of convective cores and the lower turning points when we assume $f_0 = 2$. The value $f_0 = 2$ has been determined through the calibration of the model M48 for HD 49933 in Liu et al. (2014). It can be seen that the modes of models with overshooting $r_{\alpha} = 0.2$ travel outside the convective cores for the condition $f_0 = 2$ presented in Fig. 9(d). This is also consistent with the $r_{01}$ values presented in Fig. 8, so we cannot exclude models without overshooting or models with overshooting $r_{\alpha} = 0.2$. The hypothesis $f_0 = 2$ supports the conclusion that there is a small or no convective core in the star.

The characteristics of the internal structure affect the propagation of acoustic waves in the interior of stars. Therefore, an estimate of $f_0$ can be obtained from the study of acoustic waves that penetrate into the convective cores. Moreover, the turning points of $p$ modes would affect the appearance of gravity-dominated mixed modes in the centre of subgiants and red giant stars. Constraints on the structure and physical process of cores of subgiants and red giants via mixed modes have been proposed (e.g. Deheuvels & Michel 2010, 2011; Mosser et al. 2012). These works also give us a hint towards determining the positions of the lower turning points of $p$ modes via their mixture with $g$ modes of post-main-sequence stars, for further studies.

5 DISCUSSIONS AND CONCLUSIONS

We carry out data processing and numerical modelling for KIC 6225718, observed by the Kepler mission, and derive the basic stellar parameters. The main results are as follows.

(i) By using the ACF and the collapsed ACF of the power spectrum, the mean large-frequency spacing $\langle \Delta f \rangle = 105.78 \pm 0.65 \mu$Hz and the frequency of maximum power $v_{\text{max}} = 2031 \pm 21 \mu$Hz are obtained. We extract the frequencies of modes of degree $l = 0, 1, 2$ for a radial order ranging from 13–24 for KIC 6225718 via the MCMC algorithms.

(ii) A grid of models with core overshooting is constructed with the YREC code, under both asteroseismic and non-asteroseismic constraints; the best-fitting models are selected. All models are
Figure 9. Comparison of the turning points of $l = 1$ modes and the radii of convective cores. The triangle presents the turning points of $l = 1$ modes and the horizontal red dotted, dashed and long-dashed lines denote the radii of convective cores of models with 1.11, 1.12 and 1.13 $M_\odot$. We notice that there are no convective cores for models without overshooting. The values of $r_t$ obtained by adopting $f_0 = 1$ and $f_0 = 2$ in equation (16) are shown in the left and right panels, respectively.

at the stage of H-core burning, which indicates that the star is at the phase of the main sequence. From the best-fitting models, we obtained $M = 1.10^{+0.08}_{-0.06} M_\odot$, $R = 1.22^{+0.01}_{-0.00} R_\odot$ and $t = 3.35^{+0.36}_{-0.75}$ Gyr for KIC 6225718. The mass and radius in this work are slightly lower than those from Silva Aguirre et al. (2012) and Huber et al. (2012), but our parameters may be more reliable because we used individual frequencies as constraints.

(iii) We propose that the upper limit of the overshooting parameter is approximately 0.2 for KIC 6225718, which means either a small convective core or no convective core exists in the star. For an F-type star with 1.1 $M_\odot$, the acoustic behaviour would not be affected significantly by the convective core and thus we cannot detect the existence of a convective core or measure its size by asteroseismology. For an F-type star with a mass higher than 1.15 $M_\odot$, the larger convective core could be detected when the acoustic waves are affected by the convective core.

(iv) There are still some discrepancies between the theoretical and observed frequencies on the echelle diagram. These may result from near-surface effects or from misunderstanding internal physical processes. Further works that consider the use of near-surface terms and inclusion of the effects of rotation and magnetic field are needed to solve the discrepancies.

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