Period spacings in red giants

II. Automated measurement

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ABSTRACT

Context. The space missions CoRoT and Kepler have provided photometric data of unprecedented quality for asteroseismology. A very rich oscillation pattern was discovered for red giants, including mixed modes that are used to decipher the red giants interiors. They carry information on the radiative core of red giants stars and bring strong constraints on stellar evolution.

Aims. Since more than 15,000 red giant light curves were observed by Kepler, we aim at developing a simple and efficient method for automatically characterizing the mixed-mode pattern and measuring the asymptotic period spacing.

Methods. With the asymptotic expansion of the mixed modes, we have revealed the regularity of the gravity-mode pattern. The stretched periods were used to study the evenly-space periods with a Fourier analysis and to measure the gravity period spacing, even when rotation severely complicates the oscillation spectra.

Results. We automatically measured gravity period spacing for more than five thousand Kepler red giants. The results confirm and extend previous measurements made by semi-automated methods. We also unveil the mass and metallicity dependence of the relation between the frequency spacings and the period spacings for stars on the red giant branch.

Conclusions. The delivery of thousands of period spacings combined with all other seismic and non-seismic information provides new bases for detailed ensemble asteroseismology.

Key words. Stars: oscillations – Stars: interiors – Stars:evolution – Methods: data analysis

1. Introduction

After the data provided by the CoRoT satellite and four years of observation of the space mission Kepler, many important studies were carried out (De Ridder et al. 2009; Bedding et al. 2010; Beck et al. 2011; Bedding et al. 2011; Beck et al. 2012; Mosser et al. 2012b). The observed pulsations correspond mostly to pressure modes which are the signature of acoustic waves stochastically excited by turbulent convection in the outer layers of the star. For red giants, the radial pressure mode pattern is now understood in a canonical form, called universal red giant oscillation pattern (Mosser et al. 2011b), which includes the asymptotic contribution of the rapid variation of the sound speed at the second helium ionization zone (Vrard et al. 2015). In combination with effective temperatures, the information derived from the radial modes is used to deliver unique formation on the stellar masses and radii (e.g., Kallinger et al. 2010).

Red giant oscillation spectra also exhibit mixed modes. They were identified in red giants by Beck et al. (2011). As they behave as acoustic waves in the envelope and gravity waves in the core, they carry unique information on the physical conditions inside the stellar cores. Dipole mixed modes were used to distinguish core-helium burning giants (clump stars) from hydrogen-shell burning giants (RGB stars) (Bedding et al. 2011; Mosser et al. 2011a; Stello et al. 2013). Contrary to pressure modes, evenly spaced in frequency, and to gravity modes pattern, evenly spaced in period, mixed modes show a more complicate spectrum. However, their oscillation pattern can be asymptotically described (Unno et al. 1989; Mosser et al. 2012c; Jiang & Christensen-Dalsgaard 2014). This description is based on the asymptotic period spacing $\Delta \Pi_{1}$. The asymptotic value is defined by the integration of the Brunt-Väisälä radial profile $N_{BV}$ inside the radiative inner regions $R$. For $\ell = 1$ modes, it writes

$$\Delta \Pi_{1} = \frac{2\pi^{2}}{\sqrt{2}} \left( \int_{R} \frac{N_{BV}}{r} \, dr \right)^{-1}. \quad (1)$$

Its value is related to the size of the radiative core (Montalbán & Noels 2013).

$\ell = 1$ period spacings were used to show seismic evolutionary tracks and to distinguish the different evolutionary stages of evolved low-mass stars, from subgiants to the ascent of the asymptotic giant branch (Mosser et al. 2014). So, identifying dipolar mixed modes is of prime importance. Furthermore, it opens the way to measure differential rotation in subgiants and on the low part of the RGB (Beck et al. 2012; Deheuvels et al. 2012, 2014) and to monitor the spinning down of the core rotation on the RGB and in the red clump (Mosser et al. 2012b).

For now, the values of $\Delta \Pi_{1}$ have already been extracted manually for 1110 stars (Mosser et al. 2014). Alternatively, the method by Stello et al. (2013) provides automated estimates of the mean mixed-mode spacing but is not intended to derive an accurate measurement of $\Delta \Pi_{1}$. On the con-
trary, the method by Datta et al. (2015), specially developed for measuring asymptotic period spacings, was presented for red giant stars where rotation is negligible, but seems likely unpracticable for stars showing rotational split-
tings. Taking into account that Kepler observed more than 15 000 red giants and that the future ESA mission Plato may significantly increase this number, it is then impor-
tant to set up an automatic method to measure $\Delta \Pi_1$.

In this work, we used the result obtained by Mosser et al.
(2015) to elaborate an automated method for determining the $\Delta \Pi_1$ parameter. Basically, oscillation frequencies are turned into stretched periods that mimic the gravity period spacings since they are evenly spaced. In Section 2, we explain the method principle, based on this change of variable completed by a Fourier analysis. In Section 3, we detail the set up of the method, including the estimate of the uncertain-
ties. In Section 4, we compare our results with the previous results of Mosser et al. (2014). This comparison helped us improving and speeding up the new method. In Section 5, we apply the method to the Kepler red giant public data; we verify the structure of the seismic evolutionary tracks and unveil their mass and metallicity dependance on the RGB. Section 6 is devoted to conclusions.

2. Principle

Our aim is to deliver period spacings in an automated way. Therefore, we make use of the asymptotic properties of the period spacings presented in a companion paper (Mosser et al. 2015).

2.1. Period spacings

The observed mixed-mode frequencies of giant stars do not exhibit the same regularity as gravity modes. However, the $\Delta \Pi_1$ quantity can be retrieved from the asymptotic relation which defines the mixed-mode pattern (Mosser et al. 2012c; Goupil et al. 2013). We use the implicit relation expressed in Mosser et al. (2015)

$$
\tan \pi \nu - \nu_p = q \tan \pi \frac{1}{\Delta \Pi_1} \frac{1}{\nu} - \frac{1}{\nu_g},
$$

(2)

where $\nu_p$ and $\nu_g$ are the asymptotic frequencies of pure pressure and gravity modes, respectively. $\Delta \nu(n_p)$ is the frequency difference between two consecutive pure pressure radial modes with radial orders $n_p$ and $n_p + 1$, and $q$ is the coupling parameter between the pressure and gravity-wave patterns.

The asymptotic frequencies of pure dipole pressure modes are computed using the relation described by Mosser et al. (2011b), which is called the universal pattern,

$$
\nu_p = \left(n_p + \frac{1}{2} + \varepsilon_p + d_{01} + \frac{\alpha}{2}(n - n_{\text{max}})^2\right) \Delta \nu,
$$

(3)

where $\varepsilon_p$ is the asymptotic offset, $d_{01}$ is the small separation corresponding to the distance, in units of $\Delta \nu$, of the pure pressure dipole mode compared to the midpoint between the surrounding radial modes, $n_{\text{max}} = \nu_{\text{max}}/\Delta \nu - \varepsilon_p$ is the non-integer order at the frequency $\nu_{\text{max}}$ of maximum oscillation signal, and $\alpha$ is a term corresponding to the second order of the asymptotic expansion (Mosser et al. 2013).

Fig. 1: Precise description of the function $\zeta$ for $\Delta \Pi_1 = 70$ s, obtained with a scan of various periods with neighboring values in the range $\Delta \Pi_1(1 \pm \nu_{\text{max}} \Delta \Pi_1/2)$.

The asymptotic frequencies of pure dipole gravity modes are computed using the first-order asymptotic expansion (Tassoul 1980):

$$
\frac{1}{\nu_g} = (-n_g + \varepsilon_g) \Delta \Pi_1,
$$

(4)

with $n_g$ the radial gravity order and $\varepsilon_g$ the gravity offset. This parameter is sensitive to the stratiﬁcation near the boundary between the radiative core and the convective envelope (Provost & Berthomieu 1986).

Following Eq. (2) the period spacing $\Delta P$ between two consecutive mixed modes writes (see Deheuvels et al. (2015) and Mosser et al. (2015) for the full development)

$$
\Delta P = \zeta \Delta \Pi_1,
$$

(5)

where $\zeta$ is the function described in Goupil et al. (2013) and Deheuvels et al. (2015) for expressing the relative contribution of the inner radiative region to the mode inertia. Following Mosser et al. (2015), $\zeta$ is derived from the Equation (2). It is defined by

$$
\zeta = \left[1 + \frac{1}{q} \frac{\nu^2 \Delta \Pi_1}{\Delta \nu(n_p)} \cos^2 \pi \frac{1}{\Delta \Pi_1} \frac{1 - \nu}{\nu - \nu_g} \right]^{-1},
$$

(6)

with exactly the same parameters as in Eq. (2). Hence, following Eq. (5), $\zeta$ provides information on the nature of the mode: a value near 1 means that the mode is gravity dominated; on the contrary, pressure-dominated mixed modes correspond to local minima of $\zeta$. Jiang & Christensen-Dalsgaard (2014) describe a similar property.

Equation (5) emphasizes that the period spacings $\Delta P$ between consecutive mixed modes are not constant. As a result, the difference between $\Delta P$ and $\Delta \Pi_1$ has to be corrected for addressing the direct measurement of $\Delta \Pi_1$.

2.2. Stretching of the spectrum

The main purpose of our method is to force the $\Delta \Pi_1$ regularity to appear in the mixed-mode pattern. Since the deforation of the period spacings is expressed by $\zeta$, this function is used to modify the frequency axis of the spectrum.
In practice, each part of the frequency axis of the spectrum is stretched according to the $\zeta$ function to account for the difference expressed by the ratio $\Delta P/\Delta \Pi_1$. We therefore use the new variable $\tau$ defined by the differential equation

$$d\tau = \frac{1}{\zeta} \frac{d\nu}{\nu^2},$$

(7)

where the term $\nu^{-2}$ expresses the shift from frequencies to periods, and the term $\zeta^{-1}$ accounts for the stretching. The role of $\zeta^{-1}$ is minor in the region of gravity-dominated mixed modes, important in the region of pressure-dominated mixed modes. With Eq. (7), the spectrum is reorganized as a function of the variable $\tau$ which has the dimensions of a time.

Mathematically, the change of variable corresponds to a bijection between the frequency and period spaces, defined by

$$B : \nu_{nm} \mapsto (n_m - n_0)\Delta \Pi_1,$$

(8)

where $n_{nm}$ is the mixed-mode order and $n_0$ is an arbitrary constant. This bijection ensures that, even if the function $\zeta$ is approximate, mixed modes will be changed into a period comb with an equidistance exactly equal to the period spacing $\Delta \Pi_1$. The fact that the constant $n_0$ is arbitrary ensures that the absolute numbering of the mixed modes is not necessary to use Eq. (8). This avoids the difficulty to estimate the (negative) gravity radial orders $n_g$ when computing the mixed-mode frequencies with Eq. (2) in order to get $n_m = n_g + n_p$.

We examine in the following paragraph the properties of the function $\zeta$.

### 2.3. Properties of $\zeta$

The function $\zeta$ was already implicitly depicted in previous work (e.g., Figs. 1 of Bedding et al. 2011; Mosser et al. 2012c, but without the normalization by $\Delta \Pi_1$). Here, we propose a thorough analysis of $\zeta$ and intend to show that, even if this seems paradoxical, this function largely depends on the seismic properties of pressure modes and not of gravity modes.

The functions $\zeta$ for close values of $\Delta \Pi_1$ are very similar, as shown in Fig. 1 obtained for a typical RGB star. The pressure-mode parameters $\Delta \nu$ and $\nu_{\text{max}}$ are fixed, whereas different values of $\zeta$ are shown for different values of $\Delta \Pi_1$. This property allows us to obtain a nearly continuous function $\zeta$ for the most precise use of Eq. (7), with small modifications of the period spacing around a given value of $\Delta \Pi_1$. For the most efficient computation of $\zeta$ for a given value of $\Delta \Pi_1$, variations in the range $\Delta \Pi_1(1 \pm \nu_{\text{max}}/\Delta \Pi_1/2)$ have to be investigated (Fig. 1).

The minimum values of $\zeta$ are governed by the global seismic parameters describing the pressure mode pattern. At first order, these minimum values are located near the first-order frequencies of dipole pressure modes $(n_p + \varepsilon_p + d_{01})\Delta \nu$, where $\varepsilon_p$ is the asymptotic offset for pressure modes and $d_{01}$ is the small separation for dipole modes. $\Delta \nu$ determines the frequency difference between each minimum of $\zeta$, $\varepsilon_p$ and $d_{01}$ define the position of the dipole pressure modes (e.g., Mosser et al. 2011b), hence determine the location of these minima. A change in these parameters can potentially produce an important change in $\zeta$. However, these parameters are precisely determined from the radial

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Fig. 2: Function $\zeta(\nu)$ for different sets of $q$ and $\Delta \Pi_1$ values representative of various evolutionary stages, obtained with the method illustrated in Fig. 1. Top: Typical $\Delta \Pi_1$ values on the RGB, with different values of $q$, $\Delta \nu = 8\mu$Hz, and $\Delta \Pi_1 = 70\,s$. Compared to high $q$, low $q$ values correspond to deeper minima and $\zeta$ near to 1 for gravity-dominated mixed modes. Middle: $\Delta \nu = 8\mu$Hz, $q = 0.15$, and different values of $\Delta \Pi_1$ met either on the RGB, on in the red clump, or in the secondary clump. Even if large variations are seen in that case, the bijection (Eq. 8) ensures a correction efficient enough for iterating the value of $\Delta \Pi_1$. The minimum and maximum values of $\zeta$, respectively reached for pressure or gravity-dominated mixed modes, are plotted in dashed and dotted lines.
mode pattern and by the frequency shift $\Delta \nu$ depicted as the universal red giant oscillation pattern.

On the contrary, the function $\zeta$ hardly depends on $q$ and $\Delta \Pi_1$. As explained in Mosser et al. (2015), the coupling parameter $q$ determines the depth of the minima (Fig. 2 top). Since, this parameter does not vary much during the red giant evolution (Mosser et al. 2012c), it has little influence on $\zeta$. As for the dependence of $\zeta$ with $\Delta \Pi_1$, the scaling of $\Delta P$ to $\Delta \Pi_1$ makes the function approximately the same for each $\Delta \Pi_1$. It follows that the values of $\Delta \Pi_1$ and $q$ have only a limited impact on $\zeta$ (Fig. 2 middle).

As a result, the determination of $\Delta \nu$ and of the frequency position of the dipole pressure modes is enough providing a relevant estimate of $\zeta$. Therefore, the use of this function for stretching the oscillation spectra results in the emergence of a regularity corresponding to the $\Delta \Pi_1$ value. Even a large change in $\Delta \Pi_1$ does not modify drastically $\zeta$ (Fig. 2 bottom), so that a RGB star can be treated with a clump $\Delta \Pi_1$ value for initiating the stretching, and conversely. This property is demonstrated in Appendix A.1.

3. Automated measurement of $\Delta \Pi_1$

In this section, we detail the way to measure period spacings in a fully automated way.

3.1. Preparation of the oscillation spectrum

The first steps of the set up are based on the pressure modes. First estimates of the values of $\Delta \nu$ and $\nu_{\text{max}}$ are obtained with the envelope autocorrelation function (Mosser & Appourchaux 2009). These values are refined by using the universal pattern (Mosser et al. 2011b) in order to enhance the accuracy of the determination of $\Delta \nu$ and to precisely locate the different oscillation modes. We do not include neither radial modes in our study, since they do not exhibit mixed modes, nor quadrupole mixed modes, since they are confined near the pressure modes and do not exhibit the same pattern as dipole mixed modes. Therefore, we suppress these modes from the spectra (second panel of Fig. 3). In practice, this operation only depends on the value of the large separation: we keep part of the spectrum with a second-order reduced frequency $x_\nu$ verifying

$$ x_\nu = \frac{\nu}{\Delta \nu} - \left( n_p + \varepsilon_p + \frac{\alpha}{2} (n - n_{\text{max}})^2 \right) \in [0.06, 0.80], \quad (9) $$

where the quadratic term accounts for the second-order asymptotic expansion (Mosser et al. 2011b, 2013). In order to avoid discontinuities due to the granulation background, this operation is applied to a background-corrected spectrum (Fig. 3b). The background is determined as in Mosser et al. (2012a).

3.2. Initial values

In order to help the iterative process, we chose guess values of $\Delta \Pi_1$ agreeing with the $\Delta \Pi_1$-$\Delta \nu$ pattern evidenced by Mosser et al. (2014), depending on the evolutionary stage. For $\Delta \nu$ above 9.5 $\mu$Hz, there is no ambiguity since the only possible evolutionary status is RGB. Below, stars may be on the RGB, or in the clump, or leaving the clump. The latter case is equivalent to the clump phase, since the $\Delta \Pi_1$ shows continuous variation at the end of the clump. So, different cases were then tested for $\Delta \nu$ below 9.5 $\mu$Hz, with a guess value agreeing either with the RGB or with the other evolutionary stages.

We then stretched the oscillation spectrum using the $\zeta$ function as described in section 2 (bottom panel of Fig. 3).

3.3. Spectrum of the stretched spectrum

To retrieve the $\Delta \Pi_1$ value, we performed a Fourier transform of the new spectrum $P(\tau)$ (Fig. 4). Due to the form of the mixed-mode signal, with high amplitudes for the pressure-dominated mixed modes and low amplitudes for the gravity-dominated modes, there is no need to use a tapering function for smoothing the spectrum and reducing aliases since the distribution of the amplitudes naturally mimics a tapering function.

Regularity in the stretched spectrum results in a clear signature in its Fourier spectrum (Fig. 4). As stated above, the period signature observed is largely independent of the initial guess value of $\Delta \Pi_1$ and $q$. However, a change of these initial values will produce a small variation of the measured period signature. An iterative process provides a stable measurement of the two parameters $q$ and $\Delta \Pi_1$ after four steps only (see Appendix A.1).

When necessary, we tested the different possible evolutionary stages and kept the coherent one, when the final value of $\Delta \Pi_1$ agrees with the hypothesis on the initial value. We also measured the mixed-mode period spacing $\Delta P$ and found very good agreement.

3.4. Test with a synthetic spectrum

In order to check possible bias of the method, we performed test with synthetic low-degree oscillation spectra, using the asymptotic relations for the frequencies of radial and dipole mixed modes. Mode amplitudes were computed following Mosser et al. (2012a). The linewidths of the profile of the mixed modes, described as Lorentzians, were derived from observations. The linewidths of the radial and $\ell = 2$ modes, useless for computing the period spacing but necessary for testing the whole automated chain, were computed following Belkacem et al. (2012). We finally multiplied the mixed-mode amplitudes with a Gaussian function to take into account the amplitude difference between pressure-dominated and gravity-dominated modes. The FWHM of this Gaussian was fixed to one-fifth of the large separation to match observed spectra. The result is shown in the top part of Fig. 5.

We tested different values of $\Delta \Pi_1$, $\nu_{\text{max}}$ and $\Delta \nu$ and retrieved in each case the initial value of $\Delta \Pi_1$ with a precision much better than 0.1 % (Fig. 5).

3.5. Performance and uncertainties

3.5.1. Confidence level

To define the confidence level, we measured the mean high-frequency noise present in the spectrum and used this value to normalize the Fourier spectrum $S$ of the stretched spectrum $P(\tau)$. In order to estimate the relevance of the detection, we assumed that the statistics of $S$ follows a $\chi^2$ distribution. This is not strictly the case, due to the rescaling from the frequency to the period domain and due to
the stretching of the spectrum induced by the change of variable (Eq. 7). However, these deformations are limited in the frequency range around \(\nu_{\text{max}}\) so that we use a similar test as provided by the \(H_0\) hypothesis, but with a dedicated calibration.

Assuming a \(\chi^2\) distribution, the detection can be considered as reliable when the local maximum of \(S\) is larger than ten times the mean noise level; the detected value then corresponds to a period signature rejecting the \(H_0\) hypothesis corresponding to pure noise with more than 99.9\% confidence (Mosser & Appourchaux 2009). Simulations, consisting in retrieving the oscillation signal in a high signal-to-noise spectrum corrupted with white noise, showed that the detection is relevant with the threshold level previously mentioned fixed at the value 13.

3.5.2. Uncertainties

The precision that can be achieved for the measurement of \(\Delta \Pi_1\) depends on the time resolution of the Fourier spectrum \(S\) of the stretched spectrum. This resolution, related to the properties of the oscillating signal, expresses as

\[
\delta(\Delta \Pi_1)_{\text{res}} = \nu_{\text{max}} \Delta \Pi_1^2,
\]

as derived in Appendix A.2. It then provides a quantitative basis for estimating reliable uncertainties.

### Table 1: Estimates of the different uncertainties for a typical RGB star with \(\Delta \nu = 8\,\mu\text{Hz}\) and \(\nu_{\text{max}} = 75\,\mu\text{Hz}\) or for a typical clump star with \(\Delta \nu = 4\,\mu\text{Hz}\) and \(\nu_{\text{max}} = 35\,\mu\text{Hz}\)

<table>
<thead>
<tr>
<th></th>
<th>RGB</th>
<th>chump</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \Pi_1) (s)</td>
<td>75</td>
<td>300</td>
</tr>
<tr>
<td>(\delta(\Delta \Pi_1)_{\text{res}}) (s)</td>
<td>0.5</td>
<td>3.1</td>
</tr>
<tr>
<td>(\delta(\Delta \Pi_1)_{\text{order}}) (s)</td>
<td>0.5</td>
<td>3.1</td>
</tr>
<tr>
<td>(\delta(\Delta \Pi_1)_{\text{alias}}) (s)</td>
<td>5.1</td>
<td>27</td>
</tr>
<tr>
<td>(\max(S))</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>(\delta(\Delta \Pi_1)_{\text{over}}) (s)</td>
<td>0.05</td>
<td>0.02</td>
</tr>
</tbody>
</table>

We investigated three different cases, depending on the mixed-mode pattern density. If the mixed-mode pattern is dense, with many gravity-dominated mixed modes, then the precision on the measurement is high since the function \(S\) can be oversampled (top of Fig. 4). It then writes, as a function of the nominal resolution \(\delta(\Delta \Pi_1)_{\text{res}}\),

\[
\delta(\Delta \Pi_1)_{\text{over}} \simeq \frac{1.6}{A} \delta(\Delta \Pi_1)_{\text{res}}, \tag{11}
\]

where \(A\) is the maximum value of \(S\) reached at \(\Delta \Pi_1\).

However, the accuracy on \(\Delta \Pi_1\) also depends on the constant \(\varepsilon_g\) of the asymptotic gravity modes. At this stage,
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Fig. 4: Top: Power spectrum of $P(\tau)$ as a function of the period for the star KIC 1995859. $\Delta \Pi_1$ for this star is 321 s. Middle: Same figure for KIC 1868101 with $\Delta \Pi_1 = 300.6$ s. Bottom: Same figure for KIC 12507577 with $\Delta \Pi_1 = 66.3$ s.

there is no complete study on this parameter (see Provost & Berthomieu 1986, for a dedicated study), so that one cannot fix its value. An uncertainty of 1 in $\varepsilon_g$ translates into an uncertainty of one radial gravity order. As shown in Appendix A.4, the uncertainty is then

$$\delta(\Delta \Pi_1)_{\text{order}} = \delta(\Delta \Pi_1)_{\text{res}}.$$  \hspace{1cm} (12)

This means that the high statistical precision has to be tempered by our inability to determine the value of the offset $\varepsilon_g$. When only a low number of gravity dominated mixed-modes are observed, it is not possible to unambiguously measure $\Delta \Pi_1$ (second panel of Fig. 4). In this case, the uncertainty corresponds to a shift of one radial order.

For evolved RGB stars, gravity-dominated mixed modes have too high inertia, so that they cannot be observed (Grosjean et al. 2014). In such cases, the ambiguity for measuring $\Delta \Pi_1$ corresponds to a window effect. The absence of observable mixed modes in the frequency ranges close to quadrupole and radial modes yields large uncertainties. A contrario, the observation of a few mixed modes in this region is most often enough to remove any degeneracy in the solution. As explained in Appendix A.5, the frequency shift due to missing gravity-dominated mixed modes around radial and quadrupole modes is

$$\delta(\Delta \Pi_1)_{\text{alias}} \simeq n_{\text{max}} \delta(\Delta \Pi_1)_{\text{order}},$$  \hspace{1cm} (13)

Fig. 5: Top: Simulated mixed-mode spectrum. Radial and $\ell = 2$ modes are absent, as depicted in Figure 3. Bottom: Power spectrum of $P(\tau)$ derived from the asymptotic relation. The initial $\Delta \Pi_1$ value was settled at 300 s. The principal aliases observed around $\Delta \Pi_1$ are indicated by red dashed lines. Harmonics of $\Delta \Pi_1$ are also seen (green dashed lines).

and helps estimating the large uncertainty introduced by an alias mismatch. For red giants, values of $n_{\text{max}}$ are typically

350
The ∆Π₁ stars (Fig. 6). The results show a good agreement between cadence data used here. Only two cases on this sample where this kind of situation was observed. We however note that the automated method fails at about ten. Typical values of the uncertainties are given in Table 1.

4. Comparison with previous results

To test the method efficiency, we compared the results obtained to the 1110 stars where Mosser et al. (2014) have manually measured the parameter ∆Π₁. We excluded from their original data set subgiants and early red giants that have a large separation ∆ν larger than 18 µHz. The oscillation spectrum of such stars can be retrieved in short-cadence time series only and are out of reach of the long-cadence spectrum of such stars can be retrieved in short-cadence time series only and are out of reach of the long-cadence data used here.

We succeeded in deducing the ∆Π₁ for more than 600 stars (Fig. 6). The results show a good agreement between the ∆Π₁ measured either manually or automatically as shown in Fig. 7, since the relative difference is less than 2% for more than 80% of the stars, and less than 10% for more than 90% of the stars. In fact, the bump present at −10% and +10% corresponds to the window effect described in Section 3.5. We confirmed that a confusion between the different evolutionary states is very rare. We met only two cases on this sample where this kind of situation was observed.

We however note that the automated method fails at retrieving a ∆Π₁ estimate in three main cases:
- Measuring ∆Π₁ is difficult at low ∆ν, when the number of gravity-dominated mixed modes is small (Dupret et al. 2009; Grosjean et al. 2014) and when the frequency resolution is poor compared to the frequency difference between consecutive mixed modes. This is particularly true for evolved RGB stars or AGB stars.
- When mixed modes have a low visibility (Mosser et al. 2012a; García et al. 2014), only the manual inspection of such stars can provide ∆Π₁, under the condition that the mode visibility is not too small.
- In a limited number of cases, buoyancy glitches that induce a modulation of the period spacing are large enough to hamper the measurement of the mean period spacing with a Fourier analysis. Most often, the modulation is small, so that the method works, but may deliver preferably an alias of ∆Π₁.

We let the analysis of glitches to a forthcoming work.

On the contrary, we noted that the presence of rotational splitting does not affect the determination of the ∆Π₁ value. This is due to the fact that the ζ function is efficient for straightening the mixed-mode pattern even when splitted by rotation. Each azimuthal order m forms a family with evenly spaced stretches periods, with a spacing ∆Π₁,m given by Mosser et al. (2015):

\[
\Delta \Pi_{1,m} \simeq \Delta \Pi_{1} \left(1 + 2m \frac{N}{N+1} \frac{\delta \nu_{\text{rot}}}{\nu_{\text{max}}} \right), \tag{14}
\]

where \(N\), equal to \(\Delta \nu / \Delta \Pi_{1} \nu_{\text{max}}^{2}\), represents the number of gravity modes in the \(\Delta \nu\)-wide frequency range around \(\nu_{\text{max}}\), and where \(\delta \nu_{\text{rot}}\) is the maximum rotational splitting. These rotational splittings are small compared to the frequency \(\nu_{\text{max}}\), so that the correction proportional to the azimuthal order \(m\) is in fact smaller than the resolution \(\delta(\Delta \Pi_{1})_{\text{res}}\) (Eq. 10). As a consequence, the period spacings of all components of the dipole modes are close to ∆Π₁, and rotation is not an issue for measuring ∆Π₁.

In order to highlight these statements, we constructed a synthetic spectrum as described in Section 3.4 with the addition of rotational splittings obtained from Eq. (17) of Deheuvels et al. (2015). We considered the case of a star seen with an inclination angle of 45° and a star seen equator-on corresponding respectively to the observation of rotational triplets and rotational doublets. In each cases, the splitting amplitudes are considered equal. An example of a part of the synthetic spectrum is shown on the left part of Fig. 8. The power spectra of \(P(\tau)\) derived from the synthetic spectra, where rotational triplets were included, are shown on the right part of Fig. 8. Different values of ∆Π₁ and \(\delta \nu_{\text{rot}}\) have been tested. In each cases, the initial ∆Π₁ value has been retrieved with a precision better than 0.2%. The only visible signature of rotation is an apparent decrease of the power spectrum of \(P(\tau)\).
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Fig. 8: Left: zoom on a precise radial order of the simulated mixed-mode spectrum shown in Figure 5. χ² noise with two degrees of freedom has been included in the spectrum with an height corresponding to 1/50 of the height of the oscillations. Right: Power spectrum of P(τ) derived from the simulated spectra. Top: simulations without rotation. Bottom: same simulations but with the addition of rotational splittings, which appear as rotational triplets with a δν_rot equal to 0.054 µHz.

5. Treatment of Kepler red giant public data

5.1. Data

We used the public long-cadence data from *Kepler* with the maximum available length, up to the quarter Q17, corresponding to 44 months of photometric observation. Original light curves were taken from the MAST program (Fanelli et al. 2011; Fraquelli & Thompson 2014). Among the 15 000 light curves in the *Kepler* public data, we processed the set of more than 12 900 stars for which the large separation ∆ν can be reliably measured.

5.2. Gravity period spacing

We could determine ∆Π₁ for about 5 000 red giants. The other stars did not satisfy the reliability level defined in Section 3.5.1. This could be due to a low signal-to-noise ratio in the spectra or to the presence of too few g-dominated mixed modes. In some limited cases, this can be due to an incorrect identification of the radial mode pattern. Despite the various checks, there are about 45 outliers in Figure 9; they represent less than 1% of the total of the detections. We however notice that large uncertainties due to the alias problem affect about 20% of the values, especially on the RGB at low ∆ν. The data are available online, at the CDS, with a flag indicating values suspected as outliers or aliases.

The results confirm and extend previous work (Bedding et al. 2011; Mosser et al. 2012c; Stello et al. 2013; Mosser et al. 2014) in their conclusions, taking into account that, here we measure asymptotic values and not mean period spacings. We see the same characteristic features except for stars starting the ascension of the AGB identified by Mosser et al. (2014). Their absence can be explained by the low signal-to-noise ratio of oscillation spectra at low ∆ν, which then induces the rejection of the measurements; on the contrary, such data can be analyzed individually.

5.3. Mass and metallicity

The scaling relations allow the determination of the stellar masses and radii from the global seismic parameters (∆ν, ν_max) and from the effective temperature (e.g., Kallinger et al. 2010). We used the effective temperatures listed in Huber et al. (2014). The relative uncertainties on the stellar mass are of about 10-15%. The variations of ∆Π₁ along
Fig. 9: $\Delta \Pi_1$ in function of the large separation $\Delta \nu$ for the *Kepler* red giant public data. The color code indicates the stellar mass ($M_\odot$). The signatures of the RGB, the main clump and the secondary clump appear clearly. The proportion of outliers, compared to the seismic evolutionary tracks defined in Mosser et al. (2014), is less than 1%.

Table 2: Values of the period spacings $\Delta \Pi_1(M, Z)$, in seconds, for a large separation $\Delta \nu = 6 \mu$Hz, with the scaling relation $\Delta \Pi_1 \propto \Delta \nu^{0.25}$. Uncertainties are of about 0.25 s.

<table>
<thead>
<tr>
<th>$M/M_\odot$</th>
<th>1.0-1.2</th>
<th>1.2-1.4</th>
<th>1.4-1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z &lt; -0.4$</td>
<td>68.0</td>
<td>67.9</td>
<td>67.3</td>
</tr>
<tr>
<td>$-0.4 &lt; Z &lt; 0$</td>
<td>68.7</td>
<td>68.2</td>
<td>67.4</td>
</tr>
<tr>
<td>$Z &gt; 0$</td>
<td>69.1</td>
<td>68.5</td>
<td>67.5</td>
</tr>
</tbody>
</table>

The mass dependence present in the main and secondary clumps (e.g. Bedding et al. 2011; Mosser et al. 2014) is confirmed. We illustrate it in Fig. 12 with a subsample of Fig. 9, with stars from the open clusters NGC 6791 ($M_{6791} = 1.15 \pm 0.03 M_\odot$ in the RGB), NGC 6811 ($M_{6811} = 2.2 \pm 0.1 M_\odot$), and NGC 6819 ($M_{6819} = 1.61 \pm 0.04 M_\odot$). The membership of these stars is defined as in Stello et al. (2010), masses are derived from Miglio et al. (2012). These evolutionary tracks of the three clusters are close to each other on the RGB, where the mass dependence is weak, but important in the helium-burning phase. We confirm the identification of three blue stragglers in NGC 6819, already identified by Corsaro et al. (2012).

We also discovered another mass dependence present in the RGB branch. At fixed properties of the stellar core (at fixed $\Delta \Pi_1$), a high-mass RGB star has a larger $\Delta \nu$ value.
than a low-mass star. This means that, despite a more massive convective envelope, high-mass stars are more dense. This relation, observed for RGB stars with a mass below 1.6 $M_\odot$, is predicted by simulations (Stello et al. 2013), but never observed. We note as well that stars on the RGB with a mass above 1.6 $M_\odot$ exhibit a large spread around the RGB branch. This phenomenon, already noted by Mosser et al. (2014), can be related to the different physical conditions when such stars reach the RGB.

The large number of stars on the RGB with a precise measurement of $\Delta \Pi_1$ allowed us to test also the metallicity dependence of the $\Delta \Pi_1 - \Delta \nu$ relation. We observe that, at fixed properties of the core (at fixed $\Delta \Pi_1$), low metallicity stars have a large spacing $\Delta \nu$ significantly higher than more metallic stars (Fig. 11). The values of $\Delta \Pi_1(M,Z)$ for $\Delta \nu = 6 \muHz$ are given in Table 2. Uncertainties on these values are about 0.25 s, significantly less than the observed spread. The metallicity dependence is high for stars below 1.2 $M_\odot$, but negligible for stars above 1.4 $M_\odot$. This agrees with the fact that low-metallicity stars are denser. The fits in Fig. 11 were obtained assuming that the slope of the log($\Delta \Pi_1$) - log($\Delta \nu$) relation is fixed, equal to 0.25 according to the global fit.

5.4. Luminosity bump

We can also note the large spread on the $\Delta \Pi_1$ value for RGB stars with $\Delta \nu$ lower than 6.5 $\muHz$. This spread occurs where simulations predict the position of the luminosity bump (Lagarde et al. 2012). It could, then, be due to such a phenomenon.

The aliasing phenomenon complicates the result at low $\Delta \nu$ on the RGB and precludes the firm identification of the signature of the luminosity bump, as discussed above. The main aliases are present as the second branch observed under the RGB branch.

5.5. Coupling parameter

The coupling factor $q$ was adjusted along with $\Delta \Pi_1$. The results are shown in Fig. 13.

RGB stars show a smaller coupling than clump stars, as stated by Mosser et al. (2012c). Our results are similar to this study for RGB stars: the mean value is $q = 0.17 \pm 0.05$. However, the results for clump stars are higher: around $0.29 \pm 0.07$, but still in the previously estimated uncertainties. The clump stars present a stronger coupling between $p$ and $g$ modes than RGB stars.
Fig. 12: Same as Fig. 9, with red giants identified as members of three open clusters observed by Kepler: NGC 6791, triangles; NGC 6819, diamonds; NGC 6811, squares.

Fig. 13: Histogram of the coupling factor $q$. The dashed blue line corresponds to RGB stars and the continuous red line to clump stars.

Following Unno et al. (1989), the value of the coupling factor is linked to the extent of the evanescent region and is limited to $1/4$. Our results however do not verify this: we measure $q$ values significantly above $1/4$. This discrepancy comes from the fact that the formalism in Unno et al. (1989) is valid for a weak coupling only (M. Takata, private communication), which is not observed. A new development is necessary to link $q$ and the size of the evanescent zone, as stated by Jiang & Christensen-Dalsgaard (2014).

6. Conclusion

We presented a new method based on the inversion of the mixed-mode asymptotic relation for determining automatically the gravity period spacing in the mixed-mode pattern of red giants stars. The efficiency of the method derives from the asymptotic properties of the radial and dipole spectrum: the radial oscillation pattern follows the universal red giant oscillation pattern, which corresponds to a second-order asymptotic development; the dipole oscillation pattern is tightly fitted by the asymptotic expansion for mixed modes. The change of variable used to analyze the stretched periods of the modes allows us to exhibit the properties of the oscillation periods and not of period spacings.

We used this new method on the red giant Kepler public data and succeeded in deducing the gravity period spacing for about 5000 red giants. The results obtained confirmed previous measurement for such stars. We unveil as well a new mass dependence for RGB stars: higher-mass stars will have a lower period spacing. This work paves the way to the precise interpretation of the mixed-mode pattern on numerous stars; a massive measurement of rotational splittings is now possible. We note that buoyancy glitches sometimes
hamper the detection of $\Delta \Pi_1$; but such glitches deserve a specific study.

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Jiang, C. & Christensen-Dalsgaard, J. 2014, MNras, 444, 3622

Appendix A: Uncertainty on $\Delta \Pi_1$

The measurement of $\Delta \Pi_1$ depends on the characteristics of the stretched spectrum and of its spectrum. We aim first at examining how they are connected with the method and with the global seismic properties of the spectrum. The exact measurement of $\Delta \Pi_1$ presupposes as well the exact determination of the mixed-mode order, which depends on the value of the gravity offset $\delta g$. The measurement is also perturbed by the low amplitudes of gravity-dominated mixed modes. Since these modes are evenly spaced in frequency, this effect is comparable to a window effect. All these effects are examined.

A.1. Uncertainties of the stretching process

The accuracy of the stretching process is ensured by the principle of the method and the use of Eq. (7). In this equation, the difference between periods and stretched periods is due to the term $1/\zeta$, which basically ensures that there are $N+1$ modes in a $\nu/\Delta \nu$-wide interval, where $N = \Delta \nu/\Delta \Pi_1 \nu^2$ modes are expected. As a result, the relative difference between periods and stretched periods is measured by $1/N$. Apart for subgiants and on the early RGB where $N$ has small values (Mosser et al. 2014), the large values of $N$ for the RGB and clump stars considered in this work ensures an efficient iteration. In order to illustrate this, Table A.1 shows the convergence of the iteration process in the case of an RGB star with $\Delta \Pi_1 = 75 \, s$, with two initial guess values (RGB or clump). Even if the initial guess value corresponds to an incorrect determination of the evolutionary stage, the iteration process precisely converges after four steps.

A.2. Resolution of the spectrum of the stretched spectrum

We aim at examining the properties of the stretched spectrum and of its spectrum.

Since the frequency range where modes are observed is typically $\nu_{\text{max}}$, the period range of the stretched spectrum is about $1/\nu_{\text{max}}$. Then, the spectrum of this spectrum has typically a resolution $\nu_{\text{max}}$. Since the signature of the period spacing $\Delta \Pi_1$ occurs at $1/\Delta \Pi_1$, the nominal resolution expressed in the period variable is

$$\delta (\Delta \Pi_1)_{\text{res}} = \nu_{\text{max}} \Delta \Pi_1^2. \quad (A.1)$$

Typical values of the nominal resolution correspond to about 0.4s on the RGB and 3.6s in the red clump.

A.3. Uncertainties with an oversampled spectrum

The high quality of signal allows us to oversample the spectrum in order to increase the resolution. This tighter resolution is however limited by the noise. Following the same approach as in Mosser & Appourchaux (2009), and especially their equations (A.4)-(A.6), we can compare the variation of the signal peaking at amplitude $A$ to the maximum variation of a noise contribution of amplitude $b$ (both expressed in white noise units). The precise identification of the signal maximum allows us to compare the oversampled resolution $\delta (\Delta \Pi_1)_{\text{over}}$ to the nominal resolution

$$\pi A \delta (\Delta \Pi_1)_{\text{over}} \geq b \delta (\Delta \Pi_1)_{\text{res}}. \quad (A.2)$$

Considering a conservative value $b = 5$, in white noise units, we have

$$\delta (\Delta \Pi_1)_{\text{over}} \approx \frac{1.6}{A} \delta (\Delta \Pi_1)_{\text{res}}. \quad (A.3)$$

Values of $A$ above the threshold level insures a significantly tighter resolution than the nominal value $\delta (\Delta \Pi_1)_{\text{res}}$. Since $A$ may be as high as 200, the accuracy of the measurement may becomes excellent. This however supposes that the gravity offset $\delta g$ intervening in the pure gravity-mode
pattern is known. It is usually set to 0, despite the fact that
the asymptotic value is 1/4 (Tassoul 1980). Anyway, its ex-
pression can be complicate, depending on the structure of
the radiative core (Provost & Berthomieu 1986).

A.4. Uncertainties corresponding to a shift of one gravity
order

According to the asymptotic theory, the gravity mode pe-
riod are evenly spaced with a mean period spacing close to
\( \Delta \Pi_1 \). So, their frequencies express
\[
\nu = \frac{1}{n_g \Delta \Pi_1},
\]
where \( n_g \) is the gravity order. By deriving this equation, we
obtain
\[
\frac{d \nu}{\nu} = -\frac{d n_g}{n_g} - \frac{d \Delta \Pi_1}{\Delta \Pi_1}.
\]
A shift of one radial order corresponds to \( d n_g = 1 \). For a
fixed set of oscillation modes, we have \( d \nu = 0 \). It follows:
\( d \Delta \Pi_1 = \Delta \Pi_1 / n_g \). By assuming that we are near the max-
umum oscillation frequency \( \nu_{\text{max}} \) and following Eq. (A.4),
the error on \( \Delta \Pi_1 \) can be written as
\[
\delta(\Delta \Pi_1)_{\text{order}} = \nu_{\text{max}}^2 \Delta \Pi_1^2.
\]
So we note that we obtain a similar result as determined
by the resolution:
\[
\delta(\Delta \Pi_1)_{\text{order}} = \delta(\Delta \Pi_1)_{\text{res}}.
\]

A.5. Uncertainties corresponding to the window effect

The suppression of the radial and quadrupole modes we
performed possibly leads to what is called a window effect.
The removal of part of the spectrum at regular frequen-
cies produces aliases (equal to \( 1/\Delta \nu \) in period) which could
be mistakenly attributed to the real value of \( \Delta \Pi_1 \). To es-
timate the uncertainties related to this possible confusion,
we have to estimate also the frequency difference between
each mode. For that, we follow Eq. (A.5) with a fixed \( \Delta \Pi_1 \)
value (\( d \Delta \Pi_1 = 0 \)) and an order variation \( d n_g = 1 \):
\[
\frac{d \nu}{\nu} = -\frac{1}{n_g}.
\]
With \( n_g \) determined by Eq. (A.4), the value of \( d \nu \) is equal
to \( \Delta \Pi_1 \nu^2 \). It corresponds to a signature at the period
\( 1/(\Delta \Pi_1 \nu^2) \). The relative period shifts due to the aliases
are then determined by
\[
\frac{d \Delta \Pi_1}{\Delta \Pi_1} = \frac{1}{\Delta \Pi_1 \nu^2}.
\]
For frequencies close to the maximum oscillation frequency
\( \nu_{\text{max}} \), this shift translates into an uncertainty on \( \Delta \Pi_1 \) ex-
pressed by
\[
\delta(\Delta \Pi_1)_{\text{alias}} = \left( \frac{\nu_{\text{max}} \Delta \Pi_1}{\Delta \nu} \right)^2 \frac{N}{N+1} = \frac{\Delta \Pi_1}{N+1}.
\]

<table>
<thead>
<tr>
<th>Iteration step ( i )</th>
<th>( \Delta \Pi_1 ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>start value</td>
<td>80</td>
</tr>
<tr>
<td>1</td>
<td>75.30</td>
</tr>
<tr>
<td>2</td>
<td>75.018</td>
</tr>
<tr>
<td>3</td>
<td>75.0011</td>
</tr>
<tr>
<td>4</td>
<td>75.0001</td>
</tr>
<tr>
<td>target value</td>
<td>75</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
</tr>
</tbody>
</table>

This however relies on the detection of pure gravity modes.
Here, mixed modes with stretched periods behave as gravity
modes, but with an additional mode that is the pressure
mode. Hence, the correct uncertainty is reduced by a factor
\( N/(N+1) \) so that
\[
\delta(\Delta \Pi_1)_{\text{alias}} = \left( \frac{\nu_{\text{max}} \Delta \Pi_1}{\Delta \nu} \right)^2 \frac{N}{N+1} = \frac{\Delta \Pi_1}{N+1}.
\]