Full text of the document is missing.
on a comparison with dynamical properties derived from double-lined eclipsing binaries. Semi-empirical tests using halo stars have furthermore indicated that masses from scaling relations are significantly overestimated compared to expectation values for luminous metal-poor ([Fe/H] < −1) giants (Epstein et al. 2014). Population synthesis models also suggest that the observed mass distributions are shifted towards higher masses compared to predictions (Sharma et al. 2016, 2017).

Theoretical work has motivated corrections to scaling relations, for example by comparing the large frequency separation (∆ν) calculated from individual frequencies with model densities (Stello et al. 2009, White et al. 2011, Guggenberger et al. 2016, Sharma et al. 2016) or an extension of the asymptotic relation (Mosser et al. 2013). A consistent result is that ∆ν scaling relation corrections should depend on T eff, evolutionary state and metallicity. However, it is as of yet unclear whether these corrections are more important for red-giant branch or red clump stars (Miglio 2012, Sharma et al. 2016). Additionally, uncertainties in modeling the driving and damping of oscillations typically prevent theoretical tests of the ν max scaling relation, although some studies have shown encouraging results (Belkacem et al. 2011).

Despite these efforts, the validity of scaling relations as a function of metallicity and evolutionary state is poorly tested. Gaia parallaxes are dramatically improving this situation by providing a large set of distances for asteroseismic samples observed by Kepler. Initial comparisons with Gaia DR1 (TGAS) using published asteroseismic distances indicated good agreement for 20 nearby dwarfs (De Ridder et al. 2016). However, subsequent work by Silva Aguirre et al. (2017) using a sample of ≈ 60 nearby dwarfs revealed a systematic offset between TGAS and asteroseismology, in agreement with results from eclipsing binaries by Stassun & Torres (2016b) and ground-based parallaxes for dwarfs at < 25 pc (Jao et al. 2016). De Ridder et al. (2016) also found a discrepancy for ≈ 900 giants, which was explained as a systematic bias in TGAS parallaxes based on a comparison to red clump stars (Davies et al. 2017). Combined with the absence of offsets for distant Cepheids (Sesar et al. 2016), these results have been interpreted as evidence for a distance-dependent systematic error in TGAS parallaxes as large as 0.39 mas down to 2 mas (≈ 20%).

Here we use TGAS parallaxes for a large sample of 2200 Kepler stars to revisit the comparison between TGAS and asteroseismology, and to test asteroseismic scaling relations. Unlike previous studies our sample has continuous coverage from the main sequence to the red-giant branch, allowing us to compare TGAS and asteroseismology over a range of distances and evolutionary states. A companion paper describes an investigation of correlated spatial differences between TGAS and the asteroseismic Kepler sample (Zinn et al., in prep).

2. TARGET SAMPLE

Our sample consists of dwarfs, subgiants, and red giants from the APOGEE-Kepler Asteroseismic Science Consortium (APOKASC, Pinsonneault et al. 2014), supplemented with seismic detections using Kepler short-cadence data from Chaplin et al. (2014) and Huber et al. (2013a). Effective temperatures and metallicities for dwarfs and subgiants were obtained from an SPC analysis of optical high-resolution spectra obtained with the TRES spectrograph at the F. L. Whipple Observatory (Buchhave et al. 2012, 2014). The SPC analysis was performed with externally constrained asteroseismic log g values, which prevents degeneracies between T eff, log g and [Fe/H] (Torres et al. 2012, Huber et al. 2013a). For giants, we adopted ASPCAP parameters from SDSS DR13 (Holtzman et al. 2015, SDSS Collaboration et al. 2016). We furthermore collected asteroseismic parameters ν max and ∆ν from a reanalysis of the (Chaplin et al. 2014) sample using all available Kepler data for dwarfs and subgiants (Serenedi et al., in prep), and version 3.6.5 of the APOKASC catalog for giants (Pinsonneault et al., in prep). We adopted values from the SYD pipeline (Huber et al. 2009), but note that differences between asteroseismic pipelines do not affect the conclusions in this paper (see also Section 3.3.1). Finally, we collected griz photometry from the Kepler Input Catalog (KIC, Brown et al. 2011), corrected to the SDSS scale following Pinsonneault et al. (2012), 2MASS JHK, Tycho B T − V T, and TGAS parallaxes (Gaia Collaboration et al. 2016b) for Lindegren et al. (2016) for each star. Our final sample contains ≈ 440 dwarfs and subgiants as well as over 1800 red giants with asteroseismic parameters, broadband photometry, and parallaxes. Table 1 lists all observables used in this study. Unless otherwise noted, all results in this paper are based on the combination of T eff and [Fe/H] from APOGEE and SPC, as described above.

Figure 1 shows the sample in a T eff-log g diagram, with the fractional TGAS parallax uncertainty color-coded. As expected the fractional parallax uncertainty is a strong function of distance and hence evolutionary state: dwarfs and subgiants have a typical fractional uncertainty of ≈ 5%, increasing to ≈ 10% for subgiants and ≈ 50% for red clump stars. Compared to Hipparcos,
this sample increases the number of asteroseismic Kepler stars with parallaxes by a factor of $\approx 20$.

3. METHODOLOGY

3.1. Direct Method

Scaling relations for solar-like oscillations are based on the global asteroseismic observables $\nu_{\text{max}}$, the frequency of maximum power, and $\Delta \nu$, the average separation of oscillation modes with the same spherical degree and consecutive radial order. The relations are defined as follows (Kjeldsen & Bedding 1995):

$$\Delta \nu \propto \left( \frac{M}{R^3} \right)^{1/2},$$  \hspace{0.5cm} (1)

$$\nu_{\text{max}} \propto \frac{M}{R^2 \sqrt{T_{\text{eff}}}}.$$  \hspace{0.5cm} (2)

Equations (1) and (2) can be rearranged to calculate radius as follows:

$$\frac{R}{R_\odot} \approx \frac{\nu_{\text{max}, \odot}}{\nu_{\text{max}, \odot}} \left( \frac{\Delta \nu}{\Delta \nu_{\odot}} \right)^{-2} \left( \frac{T_{\text{eff}, \odot}}{T_{\text{eff}, \odot}} \right)^{1/2}. $$  \hspace{0.5cm} (3)

We used $\nu_{\text{max}, \odot} = 3090 \mu$Hz and $\Delta \nu_{\odot} = 135.1 \mu$Hz, the solar reference values for the SYD pipeline (Huber et al. 2011). Corrections for the $\Delta \nu$ scaling relation (see Section 1) were calculated using asfgrid (Sharma et al. 2016). To calculate asteroseismic distances, we combined $T_{\text{eff}}$ with the radius from equation (3) to calculate luminosity, and then used the 2MASS K-band magnitude with bolometric corrections derived by linearly interpolating $T_{\text{eff}}$, log $g$, [Fe/H] and $A_V$ in the MIST/C3K grid (Conroy et al., in prep). To estimate $A_V$ we used the 3D reddening map by Green et al. (2015), as implemented in the modest package by Bovy et al. (2016). The derived distances, extinction values and bolometric corrections were iterated until convergence.

Parallaxes can also be used to calculate luminosities (and hence radii), which can be compared to asteroseismic radii. To convert parallaxes into distances we used an exponentially decreasing volume density prior with a length scale of 1.35 kpc (Bailer-Jones 2015; Astraatmadja & Bailer-Jones 2016). In practice, we implemented a Monte-Carlo method by sampling distances following the 3D dust map, and combined this with independent random normal samples for the apparent magnitude and $T_{\text{eff}}$ to calculate radii. The resulting distributions were used to calculate the mode and 1-$\sigma$ confidence interval for radii derived from each Gaia parallax. We did not implement a more complex prior (e.g. based on synthetic stellar population) due to the difficulty of reproducing the selection function of our sample, but note that the results in this paper do not heavily depend on the choice of distance prior.

3.2. Grid Modeling

The “direct method” for determining asteroseismic distances described in the previous section has the disadvantage that it relies on a reddening map, which may contain systematic errors. We therefore calculated a second set of asteroseismic distances and TGAS radii using isochrones and synthetic photometry, which allows reddening to be treated as a free parameter. We used isochrones from the MIST database (Choi et al. 2016; Paxton et al. 2011, 2013, 2015) to calculate a grid ranging in age from 0.5 to 14 Gyr with a stepsize of 0.25 Gyr and in metallicity from −2 to +0.4 dex in stepsizes of 0.02 dex. Interpolation was performed along equal evolutionary points in age and metallicity (Dotter 2016). For each model we saved synthetic photometry in 2MASS $JHK$, Tycho $B_T-V_T$, and Sloan $gri_z$, and calculated reddened photometry in each passband for a given $V$-band extinction $A_V$ by interpolating the Cardelli et al. (1989) extinction law. Asteroseismic $\nu_{\text{max}}$ and $\Delta \nu$ values for each model were calculated using Equations (1) and (2), both with and without the $\Delta \nu$ scaling relation corrections by Sharma et al. (2016).

To infer model parameters we followed the method by Serenelli et al. (2013) to integrate over all isochrone points to derive posterior distributions given a set of likelihoods and priors. Specifically, given any combination of a set of observables $x = \{B_T-V_T, g-r, r-i, i-z, J-H, H-K, \tau_{\text{eff}}, [\text{Fe/H}], \nu_{\text{max}}, \Delta \nu\}$ and model parameters $y = \{\text{age}, [\text{Fe/H}], \text{mass}, A_V\}$, the posterior probability is:

$$p(y|x) \propto p(y)p(x|y) \propto p(y) \prod_i \exp \left( - \frac{(x_i - x_i(y))^2}{2\sigma^2_{x,i}} \right).$$  \hspace{0.5cm} (4)

The likelihood function for $\pi$ was calculated as (e.g. Bailer-Jones 2015):

$$p(\pi | d) \propto \exp \left[ - \frac{1}{2\sigma^2_d} \left( \frac{\pi}{\tilde{d}} \right)^2 \right], $$ \hspace{0.5cm} (5)

where $d$ is the model distance calculated given an absolute magnitude and $A_V$ for each model, as well as the observed $K$-band magnitude. Probability distribution functions for each stellar parameter were then obtained by weighting $p(y|x)$ by the volume which each isochrone point encompasses in mass, age, metallicity, and $A_V$, and integrating the resulting distribution along a given stellar parameter (see appendix A of Casagrande et al. 2011). For ease of computation, the integration was performed only for models within 4-$\sigma$ of the constraints set by the observables.

To calculate asteroseismic distances we used as input the spectroscopic $T_{\text{eff}}$ and [Fe/H], asteroseismic $\nu_{\text{max}}$ and $\Delta \nu$, $B_T-V_T$giJHK photometry, and a flat prior in age, resulting in posterior distributions for all stellar parameters as well as extinction and distance. To calculate TGAS radii we replaced the asteroseismic observables with the TGAS parallax $\pi$, using a flat age prior and the same distance prior as adopted in the previous section.

3.3. Validation of Seismic Distances and Gaia Radii

3.3.1. Asteroseismic Parameters

Comparisons of different methods to measure asteroseismic parameters have yielded broadly good agreement (Heck et al. 2011; Verner et al. 2011; Heck et al. 2012). The median scatter between the five methods in

20 http://waps.cfa.harvard.edu/MIST/model_grids.html
the APOKASC catalog (see Pinsoneault et al. 2014) is 0.5% in $\Delta\nu$ and 1% in $\nu_{\text{max}}$, which we added in quadrature to the formal uncertainties from the SYD pipeline (see Table 1) for the analysis described in the previous section.

To test the influence of systematic errors, we compared our asteroseismic distances calculated using the direct and grid modeling method in Figure 2a. The agreement is excellent, with median offset of 0.2% and scatter of 2.6%. To test a variety of systematic errors that could enter the asteroseismic distance calculation, we compared our distances from the direct method with distances calculated using the direct model in Figure 2a. The agreement is excellent, with median offset of 0.2% and scatter of 2.6%. To test a variety of systematic errors that could enter the asteroseismic distance calculation, we compared our distances from the direct method with distances calculated using the Bellaterra Stellar Properties Pipeline (BeSPP, Serenelli et al. 2013), the BAYesian STEllar Algorithm (BASTA, Silva Aguirre et al. 2015), as well as to literature values from the Stromgren Survey for Astroseismology and Galactic Archeology (SAGA, Casagrande et al. 2014a) and Rodrigues et al. (2014). Three of these methods (BeSPP, BASTA, SAGA) used asteroseismic input values from the same pipeline but different isochrone grids, and one method used a different asteroseismic input values and isochrone models (Rodrigues et al. 2014). The median offsets are $\approx 0.2\%$ for BeSPP, 2.3% for BASTA, 0.1% for SAGA and 1.8% for Rodrigues et al. (2014), with no strong systematic trends as a function of distance (see bottom panel of Figure 2a). We thus conclude that systematic differences between asteroseismic methods to calculate distances are of the order of a few percent.

3.3.2. Extinction

Asteroseismic distances rely on extinction corrections, which can introduce systematic errors. Figure 2b compares the extinction measured using our grid-based method with the reddening map by Green et al. (2015), as applied in our direct method. We also show extinctions from Rodrigues et al. (2014), which were derived in a similar manner to the grid-modeling estimates presented here, and values from the model by Amôres & Lépine (2005), as applied by the BeSPP pipeline. The
estimates agree well for $A_V \lesssim 0.5$ mag, with a slight systematic overestimation by up to 0.2 mag of the Green et al. (2015) reddening map for $A_V \gtrsim 0.5$ mag. This comparison demonstrates that the combination of spectroscopy, asteroseismology and Gaia has strong potential for constructing empirical 3D reddening maps, in particular when combined with asteroseismic detections in different regions of the galaxy as provided by CoRoT (Hekker et al. 2009) and K2 (Stello et al. 2017). We note that the slight bias for high extinction in Figure 2b has only a small effect on Figure 2a, since the sample is dominated by stars with low extinction. Additionally, a systematic shift of 0.2 mag in $A_V$ corresponds to an error of 0.02 mag in $A_K$, or $\lesssim 1\%$ in distance. Since distances from the direct method are the least model-dependent and more directly test the validity of scaling relations, we proceed with using these values for the remainder of the paper. We note that our main conclusions are independent of whether the direct method or the grid-modeling method is adopted.

3.3.3. Bolometric Corrections

To test the effect of systematic errors in bolometric corrections, we used the method by Stassun & Torres (2016a) to calculate bolometric fluxes by fitting spectral energy distributions to broadband photometry supplemented with a grid of ATLAS model atmospheres. We then used these bolometric fluxes with $T_{\text{eff}}$ to calculate angular diameters which, combined with TGAS parallaxes, resulted in a set of stellar radii that could be directly compared with the radii calculated from TGAS parallaxes and bolometric corrections (see Section 3.1). Figure 3 shows a comparison between the two estimates.

We observe good agreement, with a median difference of $0.7\%$ and a scatter of $\approx 3\%$, and a small systematic trend with SED radii being larger by $\approx 1\%$ for red giants ($\approx 3–10 R_\odot$). We conclude that systematic errors due to bolometric corrections are at the $\approx 1\%$ level in radius (and hence also at the $\approx 1\%$ level in distance), which is small compared to the random uncertainties of TGAS parallaxes (see Figure 1).

3.4. Code Availability

The stellar classification software tools described above as well as all data to reproduce the results of this paper (Table 2) are publicly available at https://github.com/danxhuber/isoclassify. The tools can be used to derive posterior distributions for stellar parameters and distances given any input combination of asteroseismic, astrometric, photometric and spectroscopic observables.

4. RESULTS

4.1. Parallax Comparison

Figure 4 compares parallaxes from asteroseismology with those from TGAS for all 2200 stars in our sample. We show results without $\Delta \nu$ correction applied, but note that the effects of this correction are small compared to the scatter (see Section 4.2). Qualitatively, the comparison shows good agreement over three orders of magnitude. The scatter is dominated by large TGAS uncertainties for distant, evolved stars, which cause a diagonal “edge” in the ratios (bottom panel) toward low parallax values due to TGAS data systematically scatter-
subgiants, which is surprising given that scaling relations absolute systematic offset for main-sequence stars and correspond to less evolved stars. This implies a stronger offset increases for larger parallaxes, which on average discussed in Section 3.3.1. between our distances and Rodrigues et al. (2014) dis-
results are not significantly affected by the small offset Stassun & Torres (2016b) at \( \approx \) et al. (2017) correction (which predicts a similar offset to there is also tension with the upper end of the Davies
itude \( \beta \) = 55 degrees) are significantly too large, and
39 mas using an ecliptic lati-
highly unexpected, because the giant sample is magnitude limited, we observe a lack of small parallax values from asteroseismology.
are generally thought to be more reliable for stars similar to the Sun. However, asteroseismic distances scale as \( T_{\text{eff}}^{2.5} \), which varies significantly for main-sequence and subgiant stars. Indeed, \( T_{\text{eff}} \) scales are often plagued by systematic offsets (e.g. Pinsonneault et al. 2012). In general, photometric \( T_{\text{eff}} \) scales from the infrared flux method (Casagrande et al. 2011) or open clusters (An et al. 2013) are systematically hotter than spectroscopic temperatures, although recent color-\( T_{\text{eff}} \) calibrations are consistent with or cooler than spectroscopy (Huang et al. 2015). All \( T_{\text{eff}} \) scales rely on the accuracy of interferometric angular diameters (e.g. Boyajian et al. 2012a,b, White et al. 2013), some of which have been suspected to be affected by systematic errors (Casagrande et al. 2014b). While efforts to systematically cross-calibrate angular diameters between different instruments are currently underway (e.g. Huber 2016), it is still unclear which \( T_{\text{eff}} \) scale is indeed most accurate.

To test the effect of changing the \( T_{\text{eff}} \) scale, we recalculated asteroseismic distances for dwarfs and subgiants using temperatures from the APOGEE pipeline (ASPCAP), and also using photometric \( T_{\text{eff}} \) values from the infrared flux method (IRFM, Casagrande et al. 2011) and Sloan photometry (SDSS, Pinsonneault et al. 2012) as listed in Pinsonneault et al. (2012). We note that that Pinsonneault et al. (2012) used [Fe/H] = −0.2 dex and extinction values from the KIC, which were shown to be overestimated compared to values derived from asteroseismology and spectroscopy (Rodrigues et al. 2014). Accounting for these differences would result in shifts of \( \approx -20 \) K for the SDSS and \( \approx -65 \) K for the IRFM scales, depending on the adopted initial \( T_{\text{eff}} \) and extinctions. Furthermore, the SDSS and IRFM scales are not entirely independent, since SDSS was calibrated to match IRFM for \( T_{\text{eff}} > 6000 \) K. Re-deriving the SDSS and IRFM \( T_{\text{eff}} \) scales for the sample is beyond the scope of this paper, but we note that neither of these effects

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Ratio of asteroseismic and TGAS parallaxes as a function of TGAS parallax for the dwarf and subgiant sample with \( \pi > 1.5 \) mas. Colors show the logarithmic number density, with darker colors corresponding to a higher number of stars. Each panel shows a different adopted \( T_{\text{eff}} \) scale to calculate asteroseismic parallaxes. The average temperature offsets are \( \Delta(T_{\text{eff}})_{\text{SDSS-ASPCAP}} \approx 220 \text{ K}, \Delta(T_{\text{eff}})_{\text{IRFM-ASPCAP}} \approx 270 \text{ K} \), \( \Delta(T_{\text{eff}})_{\text{SDSS-SPC}} \approx 110 \text{ K} \) and \( \Delta(T_{\text{eff}})_{\text{IRFM-SPC}} \approx 140 \text{ K} \). The red dashed and dotted lines show the predicted offsets from the TGAS parallax corrections by Stassun & Torres (2016b) with and without ecliptic latitude dependence, respectively.}
\end{figure}

In agreement with Davies et al. (2017) we find that the offset increases for larger parallaxes, which on average correspond to less evolved stars. This implies a stronger absolute systematic offset for main-sequence stars and subgiants, which is surprising given that scaling relations

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig5.png}
\caption{Asteroseismic versus TGAS parallaxes for stars with \( \pi < 5 \) mas. The dashed black line shows the 1:1 relation. Light blue symbols are individual stars, while thick dark blue squares show median bins spaced by 0.5 mas. The red dashed and dotted lines show the predicted offsets from the TGAS parallax corrections by Stassun & Torres (2016b) with and without ecliptic latitude dependence, respectively. The solid red line shows the predicted offset from the TGAS parallax correction by Davies et al. (2017).}
\end{figure}
significantly change the conclusions below.

For the comparison, we discarded stars with \( \pi < 1.5 \) mas to avoid the “edge” bias that arises from large uncertainty differences discussed above. The average difference between the coolest (ASPCAP) and hottest (IRFM) \( T_{\text{eff}} \) scale is \( \approx 270 \) K. The results in Figure 6 demonstrate that the hotter \( T_{\text{eff}} \) scales bring better agreement between asteroseismic and TGAS parallaxes, particularly for \( \pi \lesssim 10 \) mas. Specifically, the median offset over the whole sample reduces by more than a factor of 2 from \( 5.8 \pm 0.6\% \) for the coolest \( T_{\text{eff}} \) scale (ASPCAP) to \( 2.0 \pm 0.7\% \) for the IRFM. Figure 6 also shows the proposed corrections by Stassun & Torres (2016b) derived from eclipsing binaries. The \( -0.25\% \) correction, which was the main result of the study, provides a good match to the data for \( \pi > 5 \) mas and spectroscopic \( T_{\text{eff}} \) scales, but is overestimated for \( \pi \lesssim 5 \) mas for all \( T_{\text{eff}} \) scales. The correction including an ecliptic latitude dependence is overestimated for \( \pi \lesssim 10 \) mas for all \( T_{\text{eff}} \) scales.

In summary, our analysis demonstrates that offsets between TGAS parallaxes, asteroseismology and eclipsing binaries are likely smaller than previously reported for \( \pi \lesssim 10 \) mas (\( \gtrsim 100 \) pc), and can be at least partially compensated by systematic errors in \( T_{\text{eff}} \) scales for dwarfs and subgiants. Residual differences are small fractions rather than absolute offsets, and are \( \approx 2\% \) for the hottest \( T_{\text{eff}} \) scales (which is consistent with results for nearby asteroseismic dwarfs by Silva Aguirre et al. 2017). These results imply that previously proposed TGAS parallax corrections may be partially overestimated for \( \pi \lesssim 10 \) mas. We note that this difference is most likely due to the larger sample size used in this study, rather than systematic differences in the adopted methods or distance scales. The above results also provide empirical evidence that hotter \( T_{\text{eff}} \) scales (such as the infrared flux method) are more accurate than cooler, spectroscopic estimates. Importantly, this conclusion assumes that there are no strong systematic errors in TGAS and asteroseismic distances.

4.2. Radius Comparison

Comparing radii instead of parallaxes reduces the \( T_{\text{eff}} \) dependence (from \( T_{\text{eff}}^{2.5} \) to \( T_{\text{eff}}^{1.5} \)), and allows a more direct test of a fundamental parameter predicted by scaling relations. Figure 7 compares asteroseismic and TGAS radii for all stars with a TGAS parallax measured to better than \( 20\% \), which approximately corresponds to the limit where the distance ratios are not heavily influenced by the exponentially decreasing volume density prior (see Section 3.1) or artefacts introduced by large differences in random errors (see Section 4.1). The overall agreement is excellent, empirically demonstrating that asteroseismic radii from scaling relations without any corrections are accurate to at least \( 10\% \) for stars ranging from \( 0.8 \) to \( 10 \) \( R_{\odot} \). The color-coding in Figure 7 furthermore demonstrates that there are no strong biases in asteroseismic radii as a function of metallicity.

To illustrate this further, Figure 8 shows the ratios as a function of \( T_{\text{eff}} \), \( \log g \), \([\text{Fe/H}]\) and TGAS radius, both with and without applying the \( \Delta \nu \) correction by Sharma et al. (2016). In addition to the raw data (small symbols) we also show median bins (large symbols). We have tested that spatial correlations between asteroseismic and TGAS parallaxes (Zinn et al., in prep) do not significantly affect these median values or their uncertainties for the typical spatial separations of stars contributing to a given bin (\( \approx 1.5 \) degrees).

We observe no significant trends with metallicity for \([\text{Fe/H}] = -0.8 \) to \( +0.4 \) dex (Figure 8). Intriguingly, however, the ratios show a trend as a function of TGAS radius (Figure 8): stars near the main sequence (\( \approx 1-1.5 R_{\odot} \)) show no offset, while the seismic radii of subgiants (\( \approx 1.5-3 R_{\odot} \)) are too small by \( \approx 5-7\% \). The offset reduces for low-luminosity red giants, before increasing for high-luminosity red giants (\( \gtrsim 10 R_{\odot} \)). The \( \Delta \nu \) scaling relation correction slightly reduces these deviations (blue triangles). We confirmed that these trends are independent of the distance prior, bolometric corrections, method for calculating asteroseismic observables, or adopted \( T_{\text{eff}} \) scales. Specifically, we used the \( \nu_{\text{max}} \) and \( \Delta \nu \) values from the COR pipeline (Mosser & Appourchaux 2009) and arrived at similar results. Adopting IRFM \( T_{\text{eff}} \) values for dwarfs and subgiants (instead of the default SPC scale) reduced the offset by \( \approx 2\% \). We added this value in quadrature in the subsequent analysis to account for \( T_{\text{eff}} \)-dependent systematics.

The upturn for high-luminosity red giants (\( \gtrsim 10 R_{\odot} \)) in Figure 8 is artificially introduced by large uncertainties of TGAS radii in a magnitude-limited sample, similar to the “edge” bias for parallaxes in the bottom panel of Figure 4. The underestimated seismic radii for subgiants, however, cannot be explained by such an effect. To put this into context, Figure 9 shows the radius ratios excluding giants with \( R > 10 R_{\odot} \), and adding results from

![Fig. 7.— Comparison of radii derived using TGAS parallaxes with radii calculated from asteroseismic scaling relations for stars with \( \sigma_\pi/\pi < 0.2 \). No \( \Delta \nu \) correction was applied. Color-coding denotes the metallicity for each star. The average residual median and scatter is \( \approx 3\% \) and \( \approx 10\% \), respectively.](image-url)
Fig. 8.— Ratio of TGAS radii over asteroseismic radii as a function of \(T_{\text{eff}}\), \(\log g\) \([\text{Fe/H}]\) and TGAS radius. Small red circles and blue triangles show unbinned data with and without applying the Sharma et al. (2016) \(\Delta \nu\) scaling relation correction, respectively. Thick symbols show median binned data. Note that the upturn for large radii is an artifact due to the large uncertainty differences between both samples (see text and Figure 9).

4.3. Red-Giant Branch versus Red Clump

Models of red giants lead us to expect a systematic difference in the \(\Delta \nu\) scaling relation as a function of the evolutionary state due to the changes in their interior sound-speed profile after the onset of He-core burning (Miglio et al. 2012). However, the degree and even the sign of this difference is not yet fully settled. For example, Miglio et al. (2012) showed that applying the \(\Delta \nu\) correction to red clump stars improves the agreement with independent radii measured in clusters, while the results

<table>
<thead>
<tr>
<th>(R_{\text{Gaia}}(R_\odot))</th>
<th>(R_{\text{Gaia}}/R_{\text{seismo}})</th>
<th>(R_{\text{Gaia}}/R_{\text{seismo}, \Delta \nu_{\text{corr}}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>1.007 ± 0.038</td>
<td>1.008 ± 0.035</td>
</tr>
<tr>
<td>1.16</td>
<td>1.010 ± 0.024</td>
<td>1.014 ± 0.024</td>
</tr>
<tr>
<td>1.50</td>
<td>1.018 ± 0.023</td>
<td>1.015 ± 0.022</td>
</tr>
<tr>
<td>1.93</td>
<td>1.053 ± 0.024</td>
<td>1.036 ± 0.024</td>
</tr>
<tr>
<td>2.48</td>
<td>1.077 ± 0.024</td>
<td>1.069 ± 0.024</td>
</tr>
<tr>
<td>3.20</td>
<td>1.041 ± 0.030</td>
<td>1.031 ± 0.029</td>
</tr>
<tr>
<td>4.12</td>
<td>0.971 ± 0.030</td>
<td>0.977 ± 0.029</td>
</tr>
<tr>
<td>5.30</td>
<td>0.966 ± 0.036</td>
<td>0.986 ± 0.037</td>
</tr>
<tr>
<td>6.83</td>
<td>0.966 ± 0.035</td>
<td>1.004 ± 0.035</td>
</tr>
<tr>
<td>8.79</td>
<td>1.003 ± 0.036</td>
<td>1.038 ± 0.037</td>
</tr>
</tbody>
</table>

Note: \(R_{\text{seismo}, \Delta \nu_{\text{corr}}}\) corresponds to seismic radii derived using the \(\Delta \nu\) scaling relation correction by Sharma et al. (2016) (i.e. blue symbols in Figure 8). Uncertainties include a 2% systematic error due to different \(T_{\text{eff}}\) scales.
by Sharma et al. (2016) implied that the largest effect of the \( \Delta \nu \) correction applies for ascending RGB stars. Previous samples to empirically test scaling relations have been too small to decide this question.

TGAS parallaxes allow us to test the dependency of the scaling relation correction on evolutionary state. To separate RGB and red clump stars, we used classifications based on mixed mode period spacings by Stello et al. (2013) and Vrard et al. (2016). Figure 10 shows parallaxes (left panels) and radii (right panels) both with (bottom) and without (top) applying the \( \Delta \nu \) scaling relation correction by Sharma et al. (2016). The samples in each panel are separated into RGB (blue circles) and red clump stars (red triangles). Note that we relaxed the fractional parallax uncertainty cut to < 40\% to include more red clump stars in the sample.

While the scatter is too large to determine whether the RGB or red clump stars agree better with TGAS, there is tentative evidence that the \( \Delta \nu \) correction provides an improvement for RGB stars. Specifically, the weighted mean offset reduces from 2.9 \( \pm \) 1.1\% to 0.0 \( \pm \) 0.6\% in parallax and from 3.7 \( \pm \) 1.1\% to 1.0 \( \pm \) 0.5\% for radius. The corrections for red clump stars are negligible, as expected. We conclude that TGAS parallaxes are not precise enough to decide how the \( \Delta \nu \) correction depends on evolutionary state, but provide tentative evidence (at the \( \approx \) 2-\sigma level) that the Sharma et al. (2016) corrections improve the accuracy of seismic distances and radii.

4.4. Synergies of Gaia and Asteroseismic Distances

TGAS provides a first glimpse of the potential of Gaia to measure distances, and vast precision improvements are expected for upcoming data releases. Since asteroseismic and TGAS distances agree to within a few percent over several orders of magnitude, it is interesting to explore the complementary nature of Gaia and asteroseismology to measure distances to galactic stellar populations. To investigate this, we calculated the expected end-of-mission Gaia parallax precision for seismic \textit{Kepler} targets using the Gaia performance model.\(^2\)

\[
\sigma_\pi / \text{mas} = \sqrt{(1.631 + 680.766z + 32.732z^2)} (0.986 + (1 - 0.986)V - I_C),
\]

with

\[
z = \begin{cases} 
0.0685 & \text{for } G < 12.1 \\
10^{0.4(G - 15)} & \text{otherwise}
\end{cases}
\]

Here, \( \sigma_\pi \) is the predicted end-of-mission parallax uncertainty averaged over the sky. The Gaia \( G \)-band magnitude and Johnson-Cousins \( V-I \) color were calculated from KIC \textit{gri} photometry (Table 1) using the following relations (Jordi et al. 2006, 2010):

\[
G = \left( -0.0662 - 0.7854(g-r) - 0.2859(g-r)^2 + 0.0145(g-r)^3 \right) + g,
\]

and

\[
V-I = \begin{cases} 
0.675(g-r) + 0.364 & \text{for } g-r < 2.1 \\
1.11(g-r) - 0.52 & \text{otherwise}
\end{cases}
\]

To account for the sky-position dependency of parallax uncertainties due to the Gaia scanning law, we interpolated the recommended scaling factors\(^3\) for the ecliptic coordinates of each \textit{Kepler} target. This yielded on average \( \sim \) 28\% smaller uncertainty than the uncertainties calculated from Equation 8.

Figure 11 compares the distance uncertainty from asteroseismology to the expected end-of-mission Gaia precision for stars with asteroseismic distances from this

\(^2\) http://www.cosmos.esa.int/web/gaia/science-performancehttp://www.cosmos.esa.int/web/gaia/table-6
work, [Rodrigues et al. (2014), Casagrande et al. (2014a) and Mathur et al. (2016a)] analyzed giants only. Remarkably, asteroseismology will provide more precise distances than the best Gaia performance for stars beyond 3 kpc. This is because the asteroseismic sensitivity does not depend strongly on apparent magnitude and hence distant, high luminosity red giants still yield precisions of a few percent out to tens of kpc [Mathur et al. 2016a]. Asteroseismology will therefore be critical to extend the reach of Gaia to distant stellar populations, particularly if combined with spectroscopy, which simultaneously allows to constrain interstellar extinction (Figure 2). Current and future opportunities to detect oscillations in distant red giants outside the Kepler field include the K2 Mission [Howell et al. 2014], targets with 1-year coverage near the ecliptic poles observed by TESS [Ricker et al. 2014], red giants in the bulge observed with WFIRST [Gould et al. 2015], and red giants observed with PLATO [Rauer et al. 2014].

5. CONCLUSIONS

We presented a detailed comparison of asteroseismic scaling relations with Gaia DR1 (TGAS) parallaxes for 2200 Kepler stars spanning from the main sequence to the red-giant branch. Our main findings can be summarized as follows:

• Previously identified offsets between TGAS parallaxes and distances derived from asteroseismology and eclipsing binaries have been partially overestimated for stars beyond 100 pc ($\pi < 10$ mas) in the Kepler field. We emphasize that this is most likely due to the larger sample size used here, rather than systematic differences in the methods or distance scales in previous studies. We demonstrate that for subgiants and dwarfs the offsets can be in part compensated by adopting a hotter $T_{\text{eff}}$ scale (such as the infrared flux method) as opposed to spectroscopic temperatures. If systematics from scaling relations and TGAS parallaxes are negligible, these results would validate the IRFM as a fundamental $T_{\text{eff}}$ scale for dwarfs and subgiants. Residual systematic differences between asteroseismology and TGAS parallaxes are a constant fraction (at the $\approx 2 \%$ level) across three orders of magnitude, in line with the previously noted dependence of absolute TGAS parallax offsets with distance.

• Asteroseismic and Gaia radii agree to within $\approx 10 \%$ but reveal a systematic offset for subgiants ($\approx 1.5 - 3 R_\odot$), with seismic radii being underestimated by $\approx 5 - 7 \%$, with a $\approx 2 \%$ systematic error depending on the $T_{\text{eff}}$ scale. Our results show no significant offsets for main-sequence stars ($\lesssim 1.5 R_\odot$) and low-luminosity giants with ($R \approx 3 - 8 R_\odot$), indicating that the offsets derived from eclipsing binaries by [Gaulme et al. 2016] do not appear to extend to less evolved stars. Overall, our results demonstrate empirically that systematic errors in radii derived from scaling relations are at or below the $\approx 5 \%$ level from $\approx 0.8 - 10 R_\odot$.

• A comparison of parallaxes and radii for RGB and red clump stars shows tentative evidence (at the $\approx 2 \sigma$ level) that the $\Delta \nu$ scaling relation correction by [Sharma et al. 2016] improves the comparison to Gaia. However, the precision of TGAS parallaxes is insufficient to conclusively show whether the $\Delta \nu$ correction is more important for RGB or red clump stars.

• Our results provide no evidence for systematic errors in asteroseismic scaling relations as a function of metallicity from [Fe/H] $\approx -0.8$ to $+0.4$ dex. This provides empirical support for the use of asteroseismology to calibrate spectroscopic pipelines for characterizing exoplanet host stars (e.g. [Brewer et al. 2015]) and galactic archeology (e.g. [Valenti et al. 2016]).

• We used the Gaia performance model to predict that asteroseismic distances will remain more precise than Gaia end-of-mission data for stars beyond $\approx 3$ kpc. This highlights the complementary nature of Gaia and asteroseismology for measuring distances to galactic stellar populations.

The study presented here only gives a first glimpse of the powerful synergy between Gaia and asteroseismology. In-depth studies using individual frequency modeling using TGAS parallaxes will provide further insights into differences in distance scales and seismic fundamental parameters (e.g. [Metcalfe et al. 2017]), and new interferometry as well as spectrophotometry for dozens of seismic red giants will provide a more fundamental calibration of the scaling relation for stellar radii. Furthermore, Gaia DR2 is expected to provide parallaxes for nearly all $\approx 20,000$ oscillating Kepler stars (e.g. [Mathur et al. 2016b]), allowing scaling relation tests on unprecedented scales and studies which can combine frequency modeling and Gaia

![Figure 11](image-url)
data to test and improve interior models from the main-sequence to the red-giant branch.

ACKNOWLEDGMENTS

We thank Willie Torres and Yvonne Elsworth for helpful comments and discussions, and the entire Kepler and Gaia teams for making this paper possible. DH acknowledges support by the Australian Research Council’s Discovery Projects funding scheme (project number DE140101364) and support by the National Aeronautics and Space Administration under Grant NNX14AB92G issued through the Kepler Participating Scientist Program. AS is partially supported by grant ESP2015-66134-R (MINECO). VSA acknowledges support from VILLUM FONDEN (research grant 10118). WJC and GRD acknowledge support from the UK Science and Technology Facilities Council. RAG acknowledges the support of CNES. Funding for the Stellar Astrophysics Centre is provided by The Danish National Research Foundation (Grant agreement no.: DNRF106). SM acknowledges support from NASA grants NNX12AE17G, NNX15AF13G, and NNX14AB92G, as well as NSF grant AST-1411685.

This work has made use of data from the European Space Agency (ESA) mission Gaia (https://www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement. Funding for the Kepler Mission is provided by NASA’s Science Mission Directorate. This publication makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Center/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation. Funding for the Sloan Digital Sky Survey IV has been provided by the Alfred P. Sloan Foundation, the U.S. Department of Energy Office of Science, and the Participating Institutions. SDSS acknowledges support and resources from the Center for High-Performance Computing at the University of Utah. The SDSS web site is www.sdss.org. SDSS is managed by the Astrophysical Research Consortium for the Participating Institutions of the SDSS Collaboration including the Brazilian Participation Group, the Carnegie Institution for Science, Carnegie Mellon University, the Chilean Participation Group, the French Participation Group, Harvard-Smithsonian Center for Astrophysics, Instituto de Astrofísica de Canarias, The Johns Hopkins University, Kavli Institute for the Physics and Mathematics of the Universe (IPMU) / University of Tokyo, Lawrence Berkeley National Laboratory, Leibniz Institut für Astrophysik Potsdam (AIP), Max-Planck-Institut für Astronomie (MPIA Heidelberg), Max-Planck-Institut für Astrophysik (MPA Garching), Max-Planck-Institut für Extraterrestrische Physik (MPE), National Astronomical Observatories of China, New Mexico State University, New York University, University of Notre Dame, Observatario Nacional / MCTI, The Ohio State University, Pennsylvania State University, Shanghai Astronomical Observatory, United Kingdom Participation Group, Universidad Nacional Autónoma de México, University of Arizona, University of Colorado Boulder, University of Oxford, University of Portsmouth, University of Utah, University of Virginia, University of Washington, University of Wisconsin, Vanderbilt University, and Yale University.

REFERENCES

TABLE 1

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<th>( \mu_0 )</th>
<th>( \mu_1 )</th>
<th>( \varepsilon )</th>
<th>( \varepsilon_0 )</th>
<th>( \varepsilon_1 )</th>
<th>( \tau )</th>
<th>( \omega )</th>
<th>( \text{Parallax} )</th>
<th>( \text{Gaia Parallax} )</th>
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Notes:

- Values are rounded to two decimal places.
- \( \sin i \), \( \rho \), \( \mu_0 \), \( \mu_1 \), \( \varepsilon \), \( \varepsilon_0 \), \( \varepsilon_1 \), \( \tau \), and \( \omega \) are all in \( \text{mas} \) per year.
- \( \text{Parallax} \) and \( \text{Gaia Parallax} \) are in milliarcseconds (mas).

TABLE 2