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Anomalies in the *Kepler* Asteroseismic Legacy Project Data
A re-analysis of 16 Cyg A & B, KIC 8379927 and 6 solar-like stars

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ABSTRACT

I compare values of the frequencies, separation ratios, errors and covariance matrices from a new analysis of 9 solar-like stars with the Legacy project values reported by Lund et al and, for 16Cyg A&B and KIC 8379927, with values derived by Davies et al. There is good agreement between my results and Davies's for these 3 stars, but no such agreement with the Legacy project results. My frequencies differ from the Legacy values, there are inconsistencies in the Legacy frequency covariance matrices which are not positive definite, and the Legacy errors on separation ratios are up to 40 times larger than mine and the values and upper limits derived from the Legacy frequency covariances. There are similar anomalies for 6 other solar-like stars: frequencies and separation ratio errors disagree and 2 have non positive definite covariance matrices. There are inconsistencies in the covariance matrices of 27 the 66 stars in the full Legacy set and problems with the ratio errors for the vast majority of these stars.

Key words. stars: oscillations – asteroseismology – methods: data analysis – methods: analytical

1. Introduction

The *Kepler* Asteroseismic Legacy Project (Lund et al. 2017b) analysed 66 *Kepler* main sequence targets determining frequencies, separation ratios, error estimates and covariance matrices. From the outset of this project I queried the data (cf. Roxburgh 2015, 2016) so I developed my own mode fitting routine, applied this to the Legacy power spectra for 9 solar-like stars, and here compare my results with the Legacy project's values.

In Sects. 3 to 7 I compare my results for 3 *Kepler* targets, 16 Cyg A & B and KIC 8379927, with the Legacy values and results from independent analyses by Davies et al. (2015), Davies (2015), using Davies' power spectra. My results agree well with those of Davies et al., but do not agree with the Legacy project's values.

The Legacy frequencies are different and the error estimates on separation ratios are up to a factor 40 larger and exceed upper limits derived from covariance matrices by a similar factor. The covariance matrices are inconsistent as they have negative eigenvalues and are therefore not positive semi-definite as they should be, giving negative χ^2 when comparing frequency sets.

In Sect. 8 I compare Legacy and my results for a further 6 solar-like Legacy stars; 2 have non positive definite covariance matrices, none give good agreement on frequencies or separation errors. In Sect. 9 I inspect the covariance matrices and errors on separation ratios for all 66 Legacy targets and find similar anomalies. Something is amiss with the Legacy data.

The differences between the Legacy results and those of Roxburgh and Davies are clearly shown in Fig. 1, which compares the different frequency sets for 16 Cyg B for modes with heights greater than the background ($S/N > 1$). I also gives the χ^2 of the fits using the different error estimates. The bottom panel compares errors on the separation ratios r_{02} from all 4 analyses. The agreement between Roxburgh and Davies is up to 35 times better than between the Roxburgh and Legacy values.

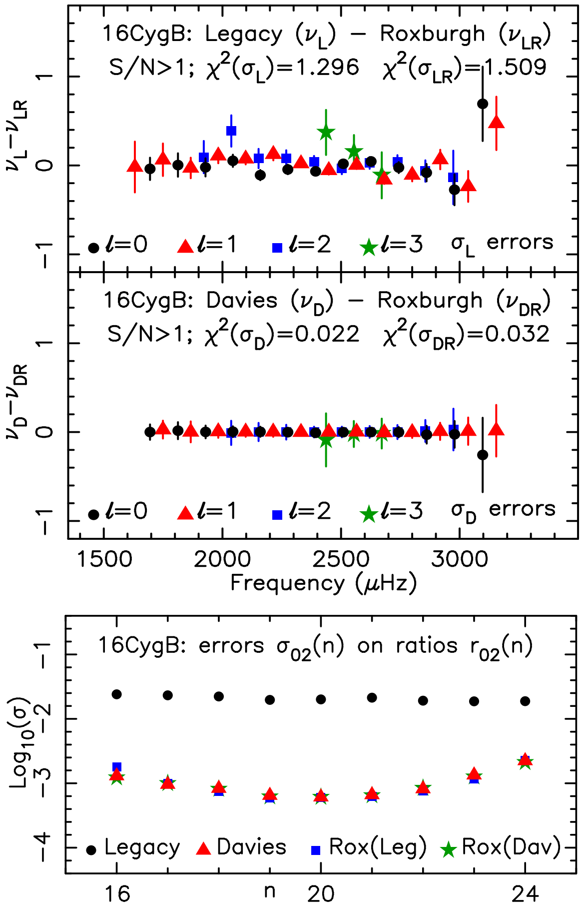

Fig. 1. 16 Cyg B: *top 2 panels*: frequency differences Legacy-Roxburgh, Davies-Roxburgh and χ^2 of fits; *bottom panel*: error estimates σ_{02} on the frequency separation ratios r_{02}, Legacy, Davies, Roxburgh.

2. Roxburgh's mode fitting algorithm

My mode fitting algorithm searches for a minimum in the negative log likelihood (cf. Toutain & Appourchaux 1994) of a global fit of mode power + background to a section of the power spectrum that extends ~300 μHz beyond both ends of the range of frequencies to be fitted, with unconstrained parameters X_k: frequencies $\nu_{n,\ell}$; mode heights h_n and widths w_n of the $\ell = 0$ modes; mode height ratios h_{10}, h_{20}, h_{30} of modes $\ell = 1, 2, 3$ to the heights of modes with $\ell = 0$ (with the geometrical constraint $1 + h_{20} = h_{10} + h_{30}$), the same for all modes; rotational splitting ν_Ω and inclination i (the same for all modes); and 4 parameters of a Harvey-like model of the background $(A/[1 + B\nu^c] + D)$. The heights and widths of the $\ell = 1, 2, 3$ modes are determined by (linear) interpolation in the values for the $\ell = 0$ modes at the respective frequencies and, for mode heights, then multiplied by the mode height ratios. The modes are fitted with symmetric rotationally split Lorentzians. The covariance matrix is the inverse of the Hessian $H(i, j) = \partial^2 MLE/\partial X_i \partial X_j$ and the errors on the X_k are given as $\sigma_k = \left[H^{-1}(k, k)\right]^{1/2}$.

Power spectra

For comparison with Davies's results I used their power spectra kindly supplied to me by Guy Davies, and for comparison with the Legacy results I used the Legacy power spectra taken from the kasoc web site namely:

Star/KIC	kasoc power spectrum	Quarters
16 Cyg A	kplr012069424_kasoc-wpsd_slc_v1.pow	Q6-17.2
16 Cyg B	kplr012069449_kasoc-wpsd_slc_v2.pow	Q6-17.2
8379927	kplr008379927_kasoc-wpsd_slc_v2.pow	Q2-17.2
9098294	kplr009098294_kasoc-wpsd_slc_v1.pow	Q5-17.2
8760414	kplr008760414_kasoc-wpsd_slc_v1.pow	Q5-17.2
6603624	kplr006603624_kasoc-psd_slc_v1.pow	Q5-17.2
6225718	kplr006225718_kasoc-wpsd_slc_v1.pow	Q6-17.2
6116048	kplr006106415_kasoc-wpsd_slc_v2.pow	Q5-17.2
6106415	kplr006106415_kasoc-wpsd_slc_v2.pow	Q6-16.3

3. Results for frequencies: 16 Cyg A & B, KIC 8379927

Tables 1 to 3 gives the χ^2 of the fits of one set of frequencies to another both for all modes and just for modes with mode-height/background $= S/N > 1$ (as determined by my fits). I used frequency errors in the fits as I encountered severe problems when using Legacy covariance matrices (see Sect. 5 below).

Table 1 compares the fit of the Legacy frequencies and errors ($\nu_\mathrm{L} \pm \sigma_\mathrm{L}$) to those of Roxburgh ($\nu_\mathrm{LR} \pm \sigma_\mathrm{LR}$) (using the Legacy power spectra), χ^2_L is the value using Legacy errors and χ^2_LR using Roxburgh's errors. χ^2_LSN is the value using Legacy errors but only comparing frequencies with $S/N > 1$, and likewise χ^2_LRSN. The first row is for the full frequency sets and the second for frequency sets with "misfits" (discussed below) removed. Table 2 gives the fit of Roxburgh's frequencies ν_DR (using Davies's power spectra) to Davies's frequencies, ν_D and Table 3 compares the Legacy and Davies's values.

The Roxburgh-Davies fit for modes with $S/N > 1$ is very good for all 3 stars, much better than that of Davies's or Roxburgh's fits to the Legacy values. The Roxburgh-Davies fit to 16 Cyg B for all frequencies is strongly influenced by the misfit of the $\nu_{14,3}$ mode which has $S/N = 0.15$ and is unreliable; the Roxburgh-Davies fit for 16 Cyg A for modes with $S/N < 1$ is strongly influenced by the $\nu_{25,0}$ mode which has $S/N = 1.08$, if this is excluded $\chi^2_\mathrm{DSN} = 0.023$, $\chi^2_\mathrm{DRSN} = 0.026$.

The frequency sets obtained from my analysis for both the Legacy and Davies power spectra, the Legacy and Davies frequencies, and my S/N values, are given in the Appendix.

Table 4 compares the rotational parameters as determined by Davies et al. and as determined by Roxburgh's fits to the Davies power spectra; there is very good agreement for all 3 stars, Roxburgh's fits to the Davies spectra yielding almost the same values as those obtained by Davies.

Table 1. χ^2 of fits of Roxburgh (ν_LR) to Legacy (ν_L) frequencies.

Star	χ^2_L	χ^2_LSN	χ^2_LR	χ^2_LRSN
16 Cyg A	0.791	0.897	7.077	8.590
16 Cyg A*	0.717	0.805	1.540	1.427
16 Cyg B	1.160	1.296	5.067	1.509
16 Cyg B†	1.155	1.296	1.412	1.509
8379927	1.121	0.427	0.776	0.499

Notes. (*) Misfits $\nu_{12,0}, \nu_{13,2}$ removed. (†) Misfit $\nu_{12,2}$ removed.

Table 2. χ^2 of fits of Roxburgh (ν_DR) to Davies (ν_D) frequencies.

Star	χ^2_D	χ^2_DSN	χ^2_DR	χ^2_DRSN
16 Cyg A	0.141	0.045	0.284	0.062
16 Cyg A*	0.127	0.023	0.263	0.026
16 Cyg B	0.167	0.022	4.227	0.032
16 Cyg B†	0.137	0.022	0.376	0.032
8379927	0.184	0.033	0.506	0.034

Notes. (*) For $S/N > 1.08$. (†) Misfits $\nu_{12,2}$ and $\nu_{14,3}$ removed.

Table 3. χ^2 of fits of Davies (ν_D) to Legacy (ν_L) frequencies.

Star	χ^2_L	χ^2_LSN	χ^2_D	χ^2_DSN
16 Cyg A	0.882	0.642	1.424	0.873
16 Cyg A*	0.784	0.642	0.949	0.873
16 Cyg B	1.617	1.496	1.786	1.910
16 Cyg B†	1.631	1.496	1.742	1.910
8379927	0.936	0.663	0.586	0.570

Notes. (*) Misfits $\nu_{12,0}, \nu_{13,2}$ removed. (†) Misfits $\nu_{12,2}$ and $\nu_{14,3}$ removed.

Table 4. Fit for rotation Davies, Roxburgh (Dspec).

	Davies		RoxD	
Star/KIC	$\nu_\Omega \sin i$	i	$\nu_\Omega \sin i$	i
16 Cyg A	0.41 ± 0.01	56 ± 6	0.40 ± 0.01	56 ± 4
16 Cyg B	0.27 ± 0.02	36 ± 12	0.27 ± 0.01	34 ± 3
8379927	1.11 ± 0.03	63 ± 6	1.11 ± 0.03	66 ± 5

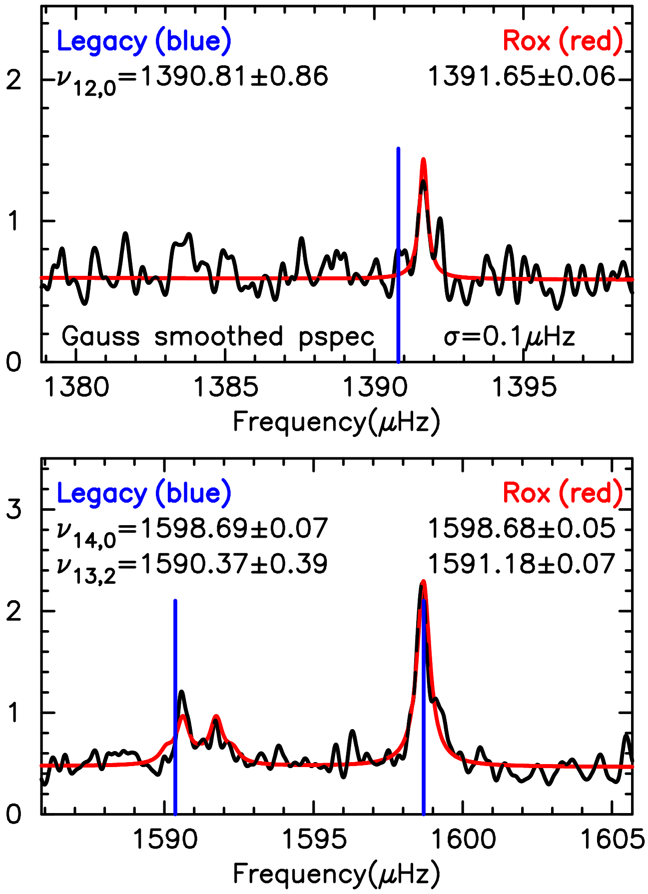

Fig. 2. 16 Cyg A: legacy and Roxburgh fits to Legacy power spectrum.

4. Fitting low frequency modes

As stated above and in the footnotes to the tables there are some problems in fitting some low frequency modes. For 16 Cyg A Legacy fits (Table 1) the problem is illustrated in Fig. 2 which shows the kasoc power spectrum for 16 Cyg A smoothed by a Gaussian smoother (with $\sigma = 0.1$ μHz), and overlaid the Roxburgh fit to the full power spectrum and the location of the Legacy and Roxburgh frequencies for modes $\nu_{12,0}$ and the pair $\nu_{13,2}, \nu_{14,0}$. The Legacy values for $\nu_{12,0}$ and $\nu_{13,2}$ are poor fits and and Roxburgh's error estimates (from the Hessian of the MLE fit) are considerably smaller than the Legacy values. Excluding these 2 modes reduces $\chi^2_{\rm LR}$ and $\chi^2_{\rm LRSN}$ from 7.077 and 8.590 to 1.540 and 1.427 respectively. A similar problem exists for the fit to $\nu_{12,2}$ for 16 Cyg B; excluding this mode reduces the $\chi^2_{\rm LR}$ of the fit using Roxburgh's errors from 5.067 to 1.412. The S/N values remain unchanged since this mode has $S/N < 1$.

Davies's value of $\nu_{12,2}$ for 16 Cyg B is also a poor fit to his power spectrum. $\nu_{14,3}$ (which has $S/N = 0.15$) differs from my value by ~3 μHz so I determined the quality of fits to the section of the Davies power spectrum between 1982.6 ± 29 μHz for a 100^2 matrix of values of $\nu_{15,1}$, $\nu_{14,3}$ and 10 values of height ratio h_{31} between 0.01 to 0.1, with fitting parameters the $\ell = 1$ mode height, one width for both $\ell = 1$ and 3 and a constant background; all with $\{\nu_\Omega \sin i, i\} = \{0.27, 34\}$. Figure 3 shows the quality of fits $(MLE - MLE_{\rm min})$ for 2 values of h_{31}; the best fits for all h_{31} have $\nu_{14,3} = 1973.71$ μHz; my full fit value is 1973.69 ± 0.37 μHz.

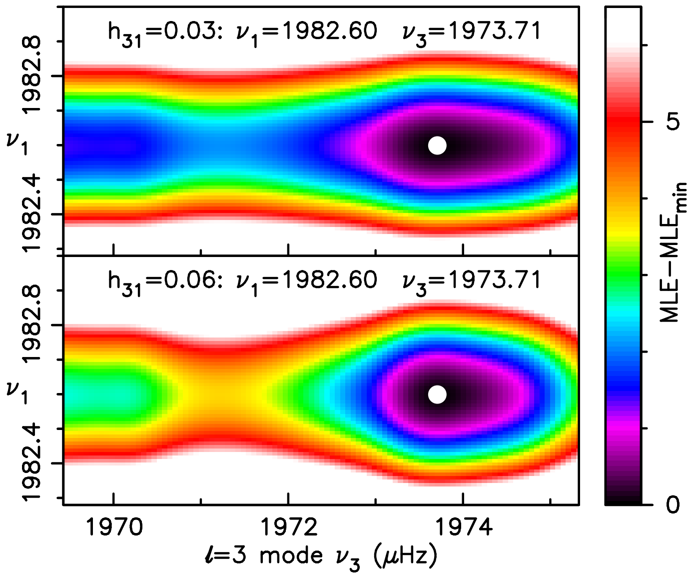

Fig. 3. 16 Cyg B: quality of free mode fits to Davies power spectrum.

Table 5. χ^2 of covariance fits of Roxburgh ($\nu_{\rm LR}$) to Legacy ($\nu_{\rm L}$).

Star	$\chi^2_{\rm L}$	$\chi^2_{\rm LSN}$	$\chi^2_{\rm LR}$	$\chi^2_{\rm LRSN}$
16 Cyg A	–3.719	0.464	7.072	8.572
16 Cyg B	–0.675	–0.432	5.071	1.511
8379927	1.293	0.211	0.797	0.504

Table 6. χ^2 of covariance fits of Davies (ν_D) to Legacy (ν_L).

Star	$\chi^2_{\rm L}$	$\chi^2_{\rm LSN}$	$\chi^2_{\rm D}$	$\chi^2_{\rm DSN}$
16 Cyg A	–0.564	0.945	1.736	0.925
16 Cyg B	4.333	0.788	1.963	2.299
8379927	–0.787	0.185	0.674	0.612

Table 7. χ^2 of covariance fits of Roxburgh ($\nu_{\rm DR}$) to Davies ($\nu_{\rm D}$).

Star	$\chi^2_{\rm D}$	$\chi^2_{\rm DSN}$	$\chi^2_{\rm DR}$	$\chi^2_{\rm DRSN}$
16 Cyg A	0.136	0.047	0.293	0.061
16 Cyg B	0.138	0,022	4.271	0.032
8379927	0.297	0.035	0.503	0.034

5. Covariance matrices and frequency comparison

The χ^2s of the fit of N frequencies incorporating their correlations are given by $[D C^{-1} D^T]/N$ where D is the vector of frequency differences and C^{-1} the inverse of the frequency covariance matrix C. Tables 5–7 give the results of such fits for 16 Cyg A & B and KIC 8379927 for both the full frequency sets and for modes with $S/N > 1$ using Legacy (L), Davies (D) and Roxburgh (R) inverse covariance matrices (determined using the SVD algorithm). Whilst the χ^2 for the Roxburgh-Davies fits are compatible (and small) and consistent with the values using frequency errors as given in Table 2, the χ^2s using the Legacy covariance matrices give negative values, which should not be the case since covariance matrices and their inverses are necessarily positive semi-definite so should always give positive χ^2.

Since a symmetric matrix C is positive semi-definite if and only if all its eigenvalues are non-negative, I determined the

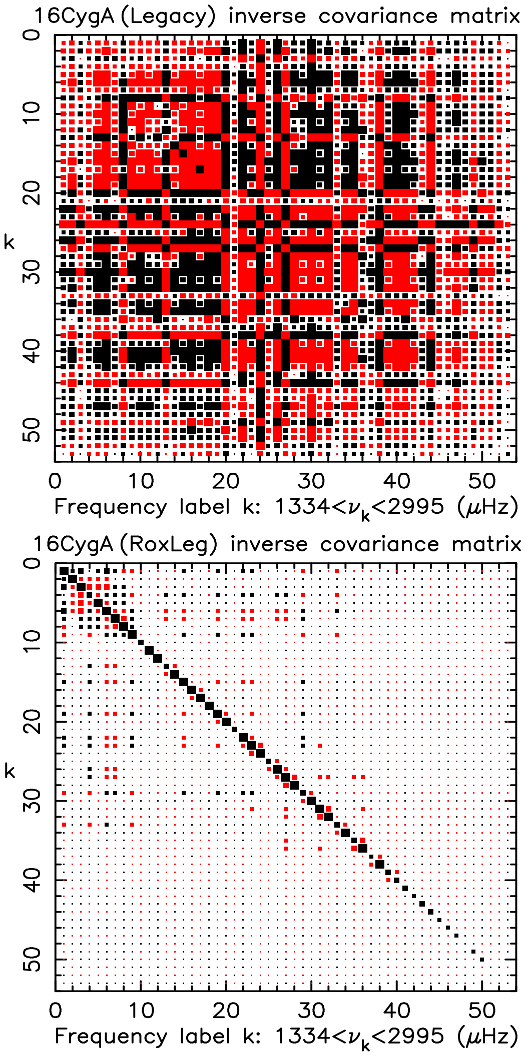

Fig. 4. 16 Cyg A: inverse covariance matrices. *Top*: legacy; *bottom*: Roxburgh [black +ve, red -ve, magnitude = size of points].

eigenvalues for the Legacy covariance matrices for all 3 stars. The absolute value of the eigenvalues w_j is given by SVD and the sign from which of $\det(C - w_j U)$ and $\det(C + w_j U)$, is zero, or closest to zero given rounding errors [U is the unit matrix]. All 3 Legacy covariance matrices have negative eigenvalues, 16 Cyg A having 10, 16 Cyg B 12, KIC 8379927 10. The Roxburgh and Davies covariance matrices are all positive definite.

The stark difference between Legacy and Roxburgh matrices is illustrated in Fig. 4 which displays their inverse covariance matrices for 16 Cyg A [magnitude=size of points, black +ve, red -ve]. Something is clearly amiss with the Legacy evaluation of the covariance matrices from their Markov chain Monte Carlo (MCMC) analysis.

6. Frequency separation ratios

The ratios of small (d) to large (Δ) frequency separations are widely used in model fitting since they are (almost) independent of the structure of the outer layers of a star. These ratios are defined as (Roxburgh & Vorontsov 2003, 2013, Roxburgh 2005)

$$r_{101}(n) = \frac{d_{101}(n)}{\Delta_n}, \quad r_{02}(n) = \frac{d_{02}(n)}{\Delta_n} \tag{1a}$$

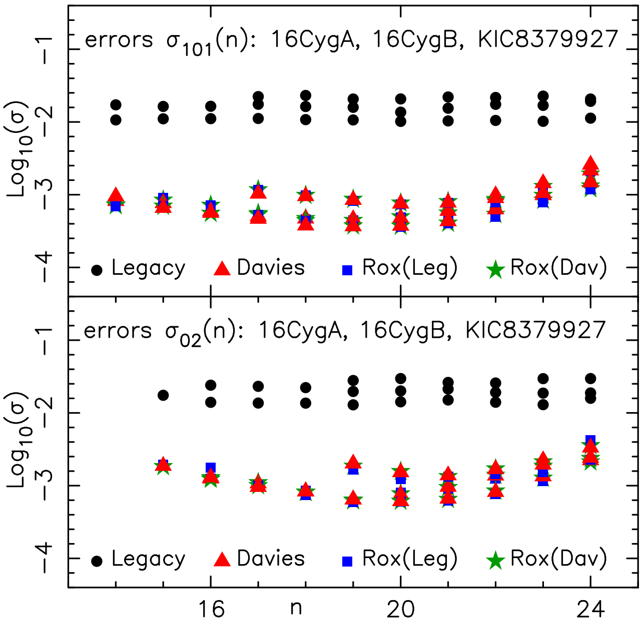

Fig. 5. *Top panel*: error estimates σ_{101} on ratios $r_{101}(n)$ from Legacy, Davies, and Roxburgh analyses of 16Cyg A&B, and KIC 8379927; *bottom errors* σ_{02} on ratios r_{02}.

where

$$d_{101}(n) = \frac{1}{8}\left[\nu_{n-1,0} - 4\nu_{n-1,1} + 6\nu_{n,0} - 4\nu_{n,1} + \nu_{n+1,0}\right] \tag{1b}$$

$$d_{02}(n) = \nu_{n,0} - \nu_{n-1,2}, \quad \text{and} \quad \Delta_n = \nu_{n,1} - \nu_{n-1.1} \tag{1c}$$

The Legacy project and Davies give values of the ratios, errors and ratio covariance matrices for the 3 stars analysed here. They also give values for r_{010} ratios but these do not contain any additional information since from 2N ($\ell = 0, 1$) frequencies one can only determine N surface layer independent quantities.

The values of the ratios r_{101} and r_{02} as determined by the different analyses are similar but, as shown in Fig. 5 the error estimates are wildly different. The top panel shows the 4 determinations of error estimates σ_{101} on the ratios r_{101} by Legacy, Davies, and Roxburgh using both the kasoc and Davies power spectra, all limited to modes with $S/N > 1$. The bottom panel shows the error estimates σ_{02} on r_{02}. Davies's and the two Roxburgh values are very close but the Legacy error estimates are very much larger than those of Davies and Roxburgh, by a factor of up to 40.

7. Error estimates and upper limits for separation ratios from frequency covariances

The covariance of two linear functions $r_n(\nu_j) = \sum A_j \nu_j$, and $r_m(\nu_k) = \sum B_k \nu_k$ of variables ν_i is given by

$$\mathrm{cov}(r_n, r_m) = \sum_j \sum_k A_j B_k \,\mathrm{cov}(\nu_j, \nu_k) \tag{2}$$

and the error estimate σ_n on $r_n(\nu_k)$ is given by the variance

$$\sigma_n^2 = \mathrm{cov}(r_n, r_n) = \sum_j \sum_k A_j A_k \,\mathrm{cov}(\nu_j, \nu_k)$$

$$= \sum_j \sum_k A_j A_k \, corr_{jk} \, \sigma_j \sigma_k \tag{3}$$

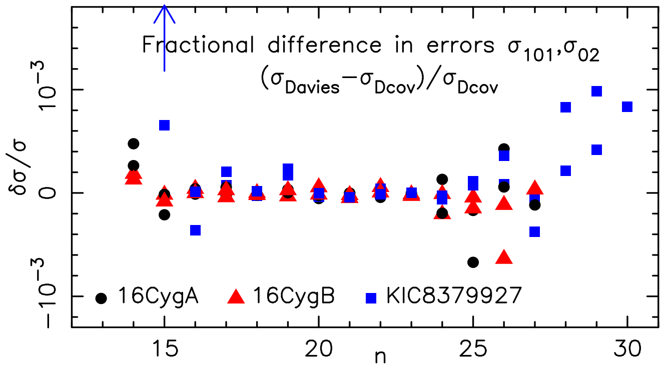

Fig. 6. Fractional difference between Davies's MCMC values for the errors σ_{101}, σ_{02} and the values from Eqs. (3), (6), (7). All but two $<10^{-3}$.

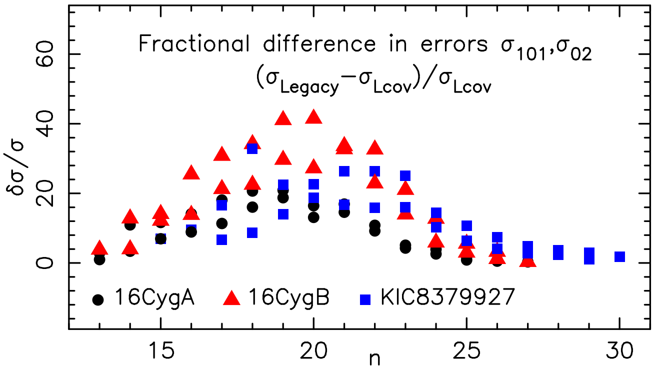

Fig. 7. Fractional difference between the Legacy MCMC values for the errors σ_{101}, σ_{02} and the $\sigma_{\rm Lcov}$ values from Eqs. (3), (6), (7).

where $corr_{jk}$ are the correlations and σ_i the error estimates on ν_i. Since $|corr_{jk}| \le 1$ it follows that an upper bound on σ_n is given by taking $corr_{jk} = +1$ if $A_j A_k > 0$ and -1 if negative, hence

$$\sigma_n \le \sigma_L, \quad \sigma_L^2 = \sum_j \sum_k |A_j A_k| \, \sigma_j \sigma_k. \qquad (4)$$

The small separations d_n [both $d_{101}(n)$ and $d_{02}(n)$] are linear functions of ν, $d_n = \sum D_k \nu_k$ (cf Eqs. (1b), (1c)), but the contribution of the large separation Δ_n introduces a small non linearity in the ratios. To a good approximation (1 in 10^3 see below) the variance of the separation ratios $r_n(\nu_k)$ is given by

$$\sigma_n^2 = \sum_j \sum_k \frac{\partial r_n}{\partial \nu_j}\frac{\partial r_n}{\partial \nu_k} \mathrm{cov}(\nu_j, \nu_k) = \sum_i \sum_j A_j A_k \, \mathrm{cov}(\nu_j, \nu_k) \qquad (5)$$

and so, with the A_k defined through Eq. (5) (see below), the error estimate σ_n on r_n is given by Eq. (3) and the upper limit $\sigma_{\rm L}$ by Eq. (4).

Coefficients A_k for errors $\sigma_{02}(n), \sigma_{101}(n)$ on $r_{02}(n), r_{101}(n)$

For $r_0 = r_{02}(n)$, $\Delta_0 = \nu_{n,1} - \nu_{n-1,1}$

$$\{\nu_k\} = \{\nu_1,\ \nu_2,\ \nu_3,\ \nu_4\} = \{\nu_{n-1,1},\ \nu_{n-1,2},\ \nu_{n,0},\ \nu_{n,1}\}$$

$$\{A_k\} = \left\{ \frac{r_0}{\Delta_0},\ -\frac{1}{\Delta_0},\ \frac{1}{\Delta_0},\ -\frac{r_0}{\Delta_0} \right\} \qquad (6)$$

For $r_0 = r_{101}(n)$, $\Delta_0 = \nu_{n,1} - \nu_{n-1,1}$

$$\{\nu_k\} = \{\nu_1, \nu_2, \nu_3, \nu_4\ \nu_5\} = \{\nu_{n-1,0}, \nu_{n-1,1}, \nu_{n,0}, \nu_{n,1}, \nu_{n+1,0}\}$$

$$\{A_k\} = \left\{ \frac{1}{8\Delta_0}, -\frac{1-2r_0}{2\Delta_0}, \frac{3}{4\Delta_0}, -\frac{1+2r_0}{2\Delta_0}, \frac{1}{8\Delta_0} \right\}. \qquad (7)$$

Figure 6 shows the fractional differences between the σ's given by Davies's analysis of 16 Cyg A & B and KIC 8379927, and the $\sigma_{\rm Dcov}$ given by Eqs. (3), (6) and (7), using Davies's frequencies and frequency covariances; all but two are less than 10^{-3}. The two are KIC 8379927 $\sigma_{02}(n)$, $n = 14, 15$, which have values -2.5×10^{-3}, 4.4×10^{-3}, and are derived from modes with $S/N < 1$.

Figure 7 shows the same comparison but between Legacy σ's and the values $\sigma_{\rm Lcov}$ derived using the Legacy frequencies and frequency covariances; here many of the differences are huge.

As shown in Fig. 8 the Legacy σ's also exceed the upper limits $\sigma_{\rm L}$ given by Eq. (4), whereas Davies's and Roxburgh's values, and the re-derived Legacy values $\sigma_{\rm Lcov}$, are less than their corresponding upper limits. Something seems to be amiss with the Legacy values.

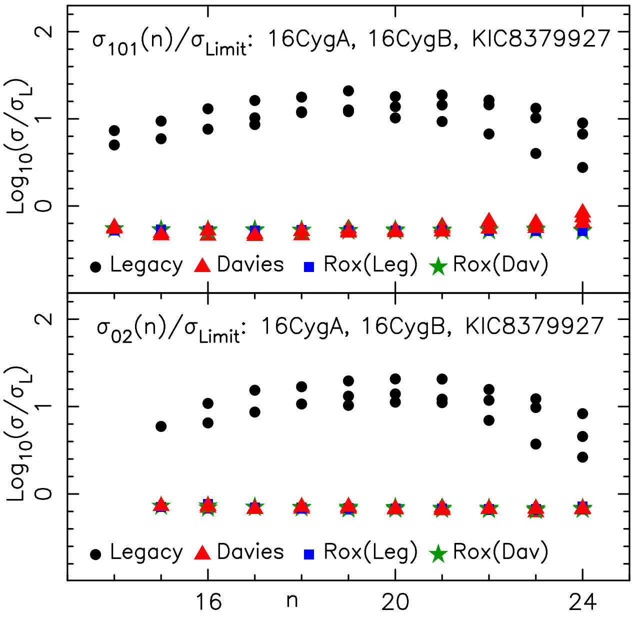

Fig. 8. Ratios of Legacy error estimates σ_{101}, σ_{02} to the upper limits $\sigma_{\rm L}$ from Eq. (4) for 16 Cyg A & B and KIC 8379927, for modes with $S/N > 1$. The Legacy values exceed the upper limits by a factor of up to 30.

8. Comparison of Legacy and Roxburgh results for a further 6 solar-like stars

Having verified that my code gives results in agreement with Davies et al., I then applied my analysis to the 6 other solar-like stars from the Legacy short list of 22 high priority targets which have large separations Δ in the range 100–120 μHz and $\nu_{\rm max}$ in the range 2138–2470 μHz, namely KIC 9098294, 8760414, 6603624, 6225718, 6116048, 6106415. The fit of the Roxburgh to Legacy frequencies for KIC 6225718 is shown in Fig. 9.

Table 8 gives the fits of the Legacy frequencies to Roxburgh's for all 6 stars using the frequency errors (rows labelled σ) and the inverse covariance matrices (labelled cov) both for all frequencies and for the subset with $S/N > 1$.

For KIC 9098294, 6603264, there is good agreement between χ^2 using the Legacy covariance matrices and uncorrelated errors, and reasonable agreement for 6225718 and 6106415, but the χ^2 are still an order of magnitude larger than the 3 Roxburgh-Davies fits for $S/N > 1$. The fits for KIC 8760414 and 6116048 are not so good: KIC 8760414 having a negative χ^2

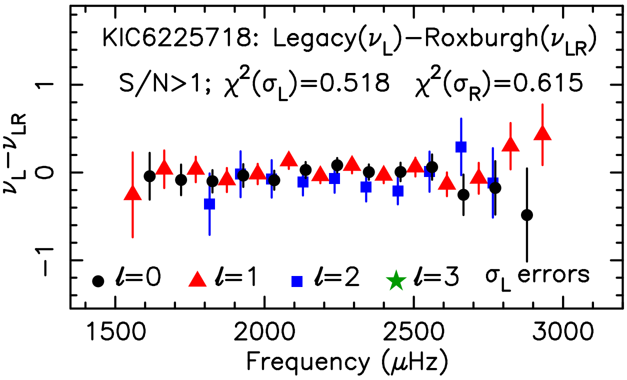

Fig. 9. KIC 6225718: frequency differences, Legacy-Roxburgh.

Table 8. Fit of Roxburgh to Legacy frequencies: 6 solar-like stars.

Star		$\chi^2_{\rm L}$	$\chi^2_{\rm LSN}$	$\chi^2_{\rm LR}$	$\chi^2_{\rm LRSN}$
9098294	σ	0.535	0.505	0.861	0.502
	cov	0.523	0.485	0.814	0.487
8760414	σ	15.170	0.355	13.462	0.380
	cov	26.301	–0.041	16.141	0.383
6603624	σ	0.762	0.226	1.967	0.268
	cov	0.794	0.225	2.001	0.268
6225718	σ	0.852	0.518	1.141	0.615
	cov	1.094	0.643	1.136	0.615
6116048	σ	0.512	0.443	0.828	0.597
	cov	0.176	0.486	0.809	0.583
6106415	σ	1.479	1.100	1.671	1.169
	cov	2.106	1.294	1.592	1.166

and KIC 6116048 a factor 3 difference between values with the Legacy covariance matrix and uncorrelated errors. Analysis of the covariance matrices revealed that for the best 4 of the 6 stars the Legacy covariance matrices had no negative eigenvalues and are therefore positive definite, whilst the other 2 have negative eigenvalues and are therefore inconsistent.

Figure 10 plots the Legacy error estimates σ_{101} and σ_{02} on the separation ratios r_{101} and r_{02} which show a similar behaviour to those of 16 Cyg A & B and KIC 8379927 in that the Legacy estimates are all larger than Roxburgh's for all 6 stars. For KIC 9098294 this is only by a factor ~2 but for KIC 6116048 the Legacy value is up to a factor 50 larger then Roxburgh's.

As was the case for 16 Cyg A & B and KIC 8379927, the Legacy ratio errors for all of these 6 stars also exceed the upper limits calculated as described in Sect. 7 above, and likewise new values for errors on the Legacy ratios calculated using the Legacy covariance matrices gave lower values, all of which are less than the corresponding upper limits.

9. Covariance matrices and errors on separation ratios for all 66 Legacy target stars

The Legacy Project analysed a total of 66 main sequence stars (Lund et al. 2017b) only 9 of which have been analysed by my code and compared with the Legacy data. Whilst this may ultimately be expanded to all the Legacy targets, I here just examine the Legacy data on all 66 stars to see whether their covariance matrices are positive semi-definite or whether they have negative eigenvalues, and whether they have anomalously large error estimates $\sigma_{101}, \sigma_{02}$ for the separation ratios r_{101}, r_{02}.

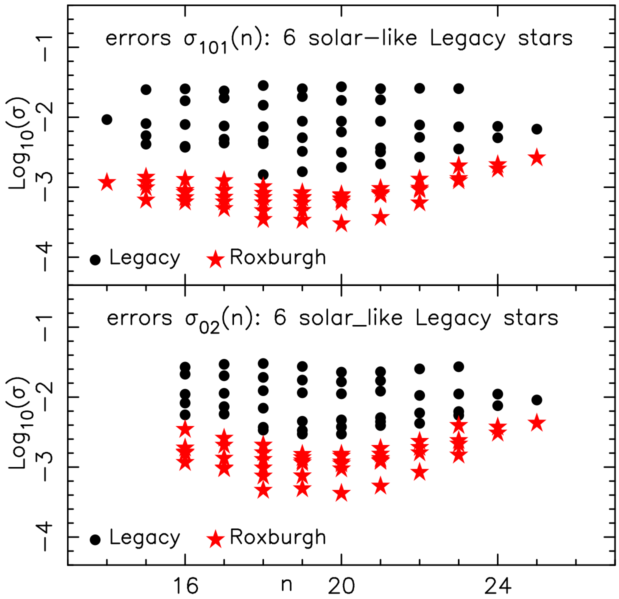

Fig. 10. Logarithm of the error estimates on the ratios r_{101}, r_{02} for all 6 solar-like stars. The Legacy values exceed Roxburgh's values by factors ranging from 2 to 50.

The eigenvalues of all 66 Legacy covariance matrices were determined by the same procedure as applied to 16 Cyg A & B and KIC 8379927, the absolute magnitudes w from SVD, and the sign from the determinants $||C \pm wU||$. 27 have covariance matrices with negative eigenvalues and are therefore inconsistent, the remaining 39 stars have positive definite covariance matrices.

Next I compare the Legacy values for the error estimates $\sigma_{101}, \sigma_{02}$ on the separation ratios r_{101}, r_{02} with the values re-derived from the frequency covariance matrices and the upper limits as determined by Eqs. (3), (4), (6), and (7) in Sect. 7. Figure 11 shows the Legacy data error estimates $\sigma_{101}, \sigma_{02}$ divided by the upper limits and the re-derived values divided by the upper limits all averaged over 3 values around their $\nu_{\rm max}$. The blue triangles are stars with inconsistent covariance matrices (negative eigenvalues). 7 stars have values of σ_{101} and σ_{02} less than their upper limits all of which have positive definite covariance matrices, of which KIC 3427720, 8938364, 9353712 and 10079226 have Legacy values for ratio errors within 2% of the re-derived values from their covariance matrices.

I selected KIC 3427720, the brightest and most solar-like of the 4 to test if, for such a star, the Legacy frequencies agreed with values obtained on applying my mode fitting algorithm to the power spectrum (kplr003427720_kasoc-wpsd_slc_v1.pow). The details of the fits are given in Table 9; as anticipated the χ^2s of the fits using errors and those using covariance matrices are in good agreement, but the values for modes with $S/N > 1$ are still more than a factor 10 larger than those of the Roxburgh-Davies fits to 16 Cyg A & B and KIC 8379927.

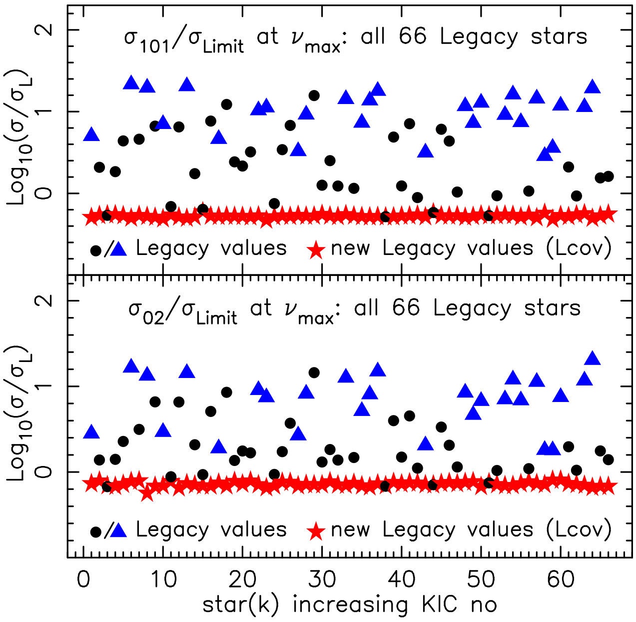

Fig. 11. Logarithm of the ratio of the error estimates to the upper limits on the ratios r_{101}, r_{02} for all 66 Legacy stars. The blue triangles are stars with inconsistent covariance matrices, the red stars the values recomputed from the covariance matrices. 59 of the Legacy values exceed one or both upper limits on σ.

Table 9. Fit of Roxburgh to Legacy frequencies: KIC 3427720.

Star		$\chi^2_{\rm L}$	$\chi^2_{\rm LSN}$	$\chi^2_{\rm R}$	$\chi^2_{\rm RSN}$
3427720	σ	0.502	0.492	0.798	0.512
	cov	0.513	0.491	0.810	0.512

10. Conclusions and discussion

1) I developed a new mode fitting code different from, and independent of, the codes used by Davies et al and the Legacy project which, when applied to the Davies et al power spectra for 16 Cyg A, 16 Cyg B and KIC 8379927, reproduces the frequencies, separation ratios, errors, rotational parameters and covariance matrices of Davies's analysis to good accuracy, especially for modes with S/N = heights/background >1 which are least sensitive to differences in the modelling of the background and the possibility of misidentification of fluctuations in noise as signal. For modes with $S/N > 1$ the χ^2 of the fits of Roxburgh to Davies's frequencies are ≤0.062, both for comparisons using only error estimates and using full covariance matrices (≤0.035 if one mode with $S/N = 1.08$ is excluded).

The same code when applied to the Legacy power spectra for 16 Cyg A, 16 Cyg B and KIC 8379927 does not reproduce the Legacy values. Frequency comparison when using covariance matrices produces anomalous results including negative χ^2; all 3 covariance matrices are inconsistent as they have negative eigenvalues and are therefore not positive semi-definite as any covariance matrix should be.

The Legacy errors on separation ratios are up to 40 times larger than my values and exceed values and upper limits derived from the Legacy frequency covariances by a similar factor.

2) I then fitted the power spectra for 6 additional solar-like stars taken from the Legacy high priority list. Here the agreement is not as bad as for 16 Cyg A & B and KIC 8739927, for modes with $S/N > 1$ the best fit of Roxburgh to Legacy (KIC 6603624) has a $\chi^2 < 0.27$ (still an order of magnitude larger than the Roxburgh-Davies fits), and good agreement between fits using errors and fits using covariance matrices; the worst fit (KIC 8760414) gave a negative χ^2 on fitting with the Legacy covariance matrix. The 4 best fits have positive definitive covariance matrices, the 2 worst fits do not. For all 6 stars the Legacy error estimates on the separation ratios exceed the values and upper limits derived using their covariances; KIC 9098294 is the one for which the Legacy values are closest to the values obtained using the frequency covariances.

3) Finally I examined all 66 Legacy targets both to test if their covariance matrices were positive semi-definite, and whether or not the errors on separation ratios satisfied their upper limits. The covariance matrices of 27 stars have negative eigenvalues and are therefore inconsistent, 39 have positive definite covariance matrices. 59 did not satisfy the upper limits on their separation ratio errors, and 4 had ratio errors consistent to within 2% of the re-derived values from their (positive definite) covariance matrices. On fitting the power spectrum of one of these, KIC 3427720; my resulting frequencies still did not agree with the Legacy values, all the fits having a $\chi^2 \sim 0.5$ whether using Roxburgh or Legacy errors or covariance matrices.

To summarise: results using my mode fitting code agree with those of Davies et al.; results from my code do not agree with the Legacy values; many of the Legacy covariance matrices are inconsistent having negative eigenvalues and therefore are not positive semi-definite; almost all of the Legacy values for errors on the separation ratios do not agree with values and upper limits derived using the Legacy covariance matrices.

It is difficult to escape the conclusion that there is something amiss with the Legacy analysis.

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Note added in proof. Lund et al. (2017a) have now produced a draft addendum to their original paper in which they have identified the reason for the overestimation of uncertainties on separation ratios as a missing trimming in their post-processing of the MCMC chains, which reduces their estimates to values comparable to the values obtained by the analysis given in this paper. They will also provide covariance matrices for the mode frequencies, separation ratios and second differences.

Lund M. N., Silva-Aguirre V., Davies G. R., et al. 2017a, manuscript published on the KASOC web site 12/06/2017

References

Davies G. R. 2015, Data distributed for analysis of KIC 8379927
Davies G. R., Chaplin W. J., Farr W. M., et al. 2015, MNRAS, 446, 2959
Lund M. N., Silva-Aguirre V., Davies G. R., et al. 2017b, ApJ, 835,172
Roxburgh, I. W. 2005, A&A, 434, 665
Roxburgh I. W. 2015, Frequency and separation ratios for the Legacy Project; Manuscript circulated to organisers of the Legacy project, 7/10/2015.
Roxburgh I. W. 2016, Proc. Seismology of the Sun and the distant stars 2016, DOI: 10.18147/smn.2016/poster:92; [`arXiv:1609.00568`]
Roxburgh, I. W., & Vorontsov S. V. 2003, A&A , 411, 215
Roxburgh, I. W., & Vorontsov S. V. 2013, A&A, 560, A2
Toutain, T., & Appourchaux T. 1994, A&A, 289, 649

Appendix A: Frequency tables

The following tables give the frequencies for 16 Cyg A, 16 Cyg B and KIC 8379927 as determined by the Legacy project, Roxburgh (Legacy), Davies, and Roxburgh (Davies), and the values of S/N from my analyses where S/N is defined as the maximum height of a rotationally split mode divided by the local background.

Table A.1. 16 Cyg A: frequencies and errors (in μHz) for Legacy, Roxburgh (Legacy), Davies, Roxburgh (Davies) and S/N values.

L	n	ν_L	σ_L	ν_{LR}	σ_{LR}	S/N	ν_D	σ_D	ν_{DR}	σ_{DR}	S/N
1	11	1334.285	1.006	1334.401	0.062	0.95	0.000	0.000	0.000	0.000	0.00
0	12	1390.808	0.863	1391.648	0.063	1.44	0.000	0.000	0.000	0.000	0.00
1	12	1437.385	0.447	1437.580	0.063	1.39	0.000	0.000	0.000	0.000	0.00
2	12	1487.831	0.707	1487.349	0.122	0.52	1488.237	0.515	1488.368	0.053	0.77
0	13	1495.053	0.235	1494.961	0.064	1.97	1495.002	0.073	1494.991	0.049	3.46
1	13	1542.060	0.140	1541.952	0.054	2.45	1541.922	0.065	1541.906	0.048	2.07
2	13	1590.366	0.387	1591.180	0.074	1.05	1591.291	0.187	1591.224	0.122	0.71
0	14	1598.690	0.072	1598.683	0.053	3.90	1598.694	0.070	1598.690	0.064	2.53
1	14	1645.140	0.109	1644.996	0.099	3.76	1645.063	0.086	1645.046	0.088	2.67
2	14	1693.937	0.186	1694.037	0.181	1.16	1694.167	0.170	1694.219	0.166	1.02
0	15	1700.952	0.101	1700.915	0.088	3.15	1700.911	0.083	1700.899	0.080	3.04
1	15	1747.199	0.085	1747.181	0.081	5.23	1747.149	0.076	1747.150	0.080	4.67
2	15	1795.843	0.131	1795.816	0.108	2.10	1795.747	0.107	1795.750	0.110	1.93
0	16	1802.351	0.079	1802.317	0.070	6.06	1802.310	0.068	1802.312	0.071	5.79
3	15	0.000	0.000	0.000	0.000	0.00	1838.516	0.669	1838.333	0.309	0.29
1	16	1849.009	0.056	1849.016	0.056	9.83	1848.976	0.053	1848.977	0.056	8.29
2	16	1898.399	0.098	1898.344	0.093	3.88	1898.262	0.099	1898.278	0.099	3.23
0	17	1904.521	0.058	1904.591	0.050	11.95	1904.609	0.058	1904.612	0.055	10.00
3	16	0.000	0.000	0.000	0.000	0.00	1941.223	0.562	1940.714	0.320	0.49
1	17	1952.008	0.050	1952.027	0.049	17.66	1951.996	0.049	1951.997	0.050	14.11
2	17	2001.588	0.082	2001.732	0.079	6.90	2001.673	0.077	2001.663	0.071	5.63
0	18	2007.538	0.045	2007.571	0.042	21.20	2007.576	0.046	2007.576	0.043	17.75
3	17	2045.851	0.368	2045.876	0.229	0.88	2045.976	0.365	2045.912	0.195	0.84
1	18	2055.493	0.047	2055.502	0.048	29.27	2055.524	0.047	2055.526	0.048	23.59
2	18	2105.374	0.056	2105.334	0.049	10.83	2105.312	0.055	2105.306	0.052	9.16
0	19	2110.949	0.041	2110.900	0.041	33.95	2110.909	0.039	2110.914	0.040	29.91
3	18	2150.057	0.204	2149.943	0.150	1.17	2149.936	0.134	2149.929	0.134	1.14
1	19	2159.149	0.049	2159.167	0.047	37.40	2159.151	0.044	2159.149	0.046	30.98
2	19	2208.928	0.072	2208.956	0.069	11.60	2208.900	0.064	2208.894	0.064	9.97
0	20	2214.225	0.054	2214.274	0.048	32.68	2214.224	0.048	2214.222	0.050	28.84
3	19	2253.796	0.250	2253.329	0.157	1.16	2253.535	0.163	2253.533	0.153	1.08
1	20	2262.562	0.051	2262.552	0.051	34.77	2262.537	0.048	2262.534	0.049	28.91
2	20	2312.505	0.079	2312.526	0.082	9.46	2312.536	0.087	2312.525	0.085	7.84
0	21	2317.282	0.057	2317.321	0.052	24.78	2317.322	0.051	2317.330	0.053	20.99
3	20	2357.497	0.227	2357.226	0.200	0.91	2357.392	0.189	2357.341	0.198	0.79
1	21	2366.245	0.060	2366.229	0.061	24.93	2366.248	0.057	2366.253	0.062	20.03
2	21	2416.249	0.123	2416.349	0.113	5.91	2416.249	0.127	2416.260	0.127	4.52
0	22	2420.937	0.080	2420.959	0.079	12.50	2420.897	0.081	2420.920	0.084	9.35
3	21	2461.452	0.358	2461.688	0.373	0.55	2462.078	0.385	2461.877	0.405	0.45
1	22	2470.227	0.091	2470.361	0.082	13.20	2470.305	0.077	2470.298	0.086	10.01
2	22	2520.734	0.199	2520.618	0.174	3.15	2520.459	0.212	2520.475	0.191	2.51
0	23	2524.950	0.156	2525.071	0.132	5.59	2525.071	0.158	2525.154	0.141	4.44
3	22	0.000	0.000	0.000	0.000	0.00	2566.969	0.608	2567.284	0.662	0.25
1	23	2574.660	0.121	2574.691	0.125	5.95	2574.784	0.126	2574.792	0.129	4.85
2	23	2624.636	0.362	2624.975	0.369	1.34	2624.322	0.324	2624.331	0.334	1.21
0	24	2628.930	0.259	2629.294	0.237	2.04	2629.204	0.178	2629.245	0.201	1.93
3	23	0.000	0.000	0.000	0.000	0.00	2669.765	1.036	2668.860	1.209	0.13
1	24	2679.726	0.201	2679.406	0.201	2.57	2679.872	0.188	2679.857	0.203	2.31
2	24	2730.024	0.756	2729.839	0.546	0.80	2730.233	0.886	2729.550	0.716	0.68
0	25	2733.571	0.420	2734.482	0.394	1.24	2733.615	0.463	2734.049	0.370	1.08
1	25	2783.816	0.335	2784.118	0.337	1.37	2784.222	0.354	2784.243	0.364	1.13
2	25	2836.088	0.798	2836.291	1.692	0.31	2835.339	1.147	2834.364	2.878	0.24
0	26	2840.148	0.944	2838.819	1.264	0.47	2838.398	0.779	2838.578	0.922	0.35
1	26	2890.198	0.692	2890.361	0.719	0.59	2891.270	0.740	2891.381	0.814	0.45
2	26	2940.393	1.103	2941.252	3.200	0.18	2941.479	1.538	2939.644	2.527	0.15
0	27	2944.937	0.792	2945.011	1.683	0.27	2945.321	1.179	2946.213	1.033	0.23
1	27	2994.840	1.013	2994.958	1.981	0.27	2996.375	1.191	2996.211	1.907	0.28

Table A.2. 16 Cyg B: frequencies and errors (in μHz) for Legacy, Roxburgh (Legacy), Davies, Roxburgh (Davies) and S/N values.

L	n	$\nu_{\rm L}$	$\sigma_{\rm L}$	$\nu_{\rm LR}$	$\sigma_{\rm LR}$	S/N	$\nu_{\rm D}$	$\sigma_{\rm D}$	$\nu_{\rm DR}$	$\sigma_{\rm DR}$	S/N
1	12	1631.088	0.286	1631.105	0.035	1.61	0.000	0.000	0.000	0.000	0.00
2	12	1685.793	0.664	1686.578	0.057	0.55	1686.419	0.313	1686.822	0.032	0.57
0	13	1695.023	0.126	1695.061	0.063	2.25	1695.069	0.087	1695.069	0.053	2.71
1	13	1749.253	0.183	1749.189	0.084	1.97	1749.214	0.101	1749.186	0.062	2.53
2	13	1804.243	0.587	1803.859	0.211	0.58	1804.168	0.273	1804.249	0.170	0.51
0	14	1812.444	0.133	1812.440	0.068	1.89	1812.428	0.097	1812.412	0.078	1.59
1	14	1866.483	0.117	1866.511	0.092	2.67	1866.523	0.118	1866.521	0.098	2.50
2	14	1921.246	0.181	1921.152	0.139	1.00	1921.206	0.160	1921.194	0.162	0.91
0	15	1928.886	0.103	1928.908	0.076	2.86	1928.901	0.072	1928.899	0.070	2.53
3	14	0.000	0.000	0.000	0.000	0.00	1970.959	5.137	1973.695	0.375	0.15
1	15	1982.607	0.084	1982.498	0.073	4.41	1982.592	0.071	1982.586	0.072	4.60
2	15	2037.203	0.177	2036.815	0.192	1.63	2036.667	0.137	2036.676	0.128	1.72
0	16	2044.357	0.069	2044.305	0.067	4.44	2044.278	0.060	2044.273	0.058	4.97
3	15	0.000	0.000	0.000	0.000	0.00	2085.370	1.498	2085.478	0.353	0.24
1	16	2098.163	0.064	2098.087	0.058	7.20	2098.084	0.057	2098.081	0.058	7.19
2	16	2152.517	0.109	2152.440	0.098	2.68	2152.420	0.102	2152.419	0.099	2.47
0	17	2159.503	0.058	2159.612	0.061	7.40	2159.581	0.057	2159.577	0.059	6.35
3	16	0.000	0.000	0.000	0.000	0.00	2200.579	1.224	2200.453	0.354	0.37
1	17	2214.334	0.069	2214.208	0.056	12.41	2214.166	0.056	2214.163	0.058	11.31
2	17	2269.112	0.094	2269.034	0.073	4.76	2268.956	0.083	2268.957	0.083	4.37
0	18	2275.949	0.054	2275.994	0.049	13.13	2275.948	0.048	2275.949	0.047	11.44
3	17	2318.958	0.290	2318.917	0.230	0.78	2319.120	0.374	2319.208	0.214	0.67
1	18	2331.163	0.041	2331.141	0.042	23.45	2331.138	0.040	2331.139	0.043	21.01
2	18	2386.252	0.070	2386.214	0.057	8.82	2386.263	0.061	2386.262	0.060	7.66
0	19	2392.645	0.042	2392.711	0.041	26.87	2392.711	0.043	2392.719	0.040	22.21
3	18	2436.781	0.255	2436.409	0.250	1.19	2436.656	0.299	2436.744	0.194	1.04
1	19	2448.181	0.048	2448.237	0.041	35.07	2448.253	0.041	2448.251	0.042	32.20
2	19	2503.411	0.066	2503.444	0.059	11.41	2503.498	0.060	2503.497	0.059	10.54
0	20	2509.678	0.042	2509.659	0.040	33.15	2509.667	0.041	2509.668	0.040	29.09
3	19	2554.181	0.188	2554.026	0.125	1.41	2554.146	0.147	2554.167	0.157	1.25
1	20	2565.426	0.043	2565.422	0.042	40.07	2565.403	0.042	2565.400	0.042	37.08
2	20	2620.562	0.066	2620.534	0.059	12.00	2620.564	0.066	2620.562	0.062	10.89
0	21	2626.458	0.050	2626.413	0.045	32.90	2626.397	0.045	2626.397	0.044	28.60
3	20	2671.592	0.260	2671.703	0.174	1.32	2671.722	0.168	2671.738	0.167	1.11
1	21	2682.247	0.047	2682.407	0.048	34.54	2682.402	0.048	2682.407	0.049	29.52
2	21	2737.707	0.075	2737.666	0.073	8.74	2737.744	0.079	2737.743	0.080	7.02
0	22	2743.322	0.062	2743.346	0.054	20.06	2743.329	0.058	2743.330	0.060	14.41
3	21	2789.000	0.365	2788.887	0.250	0.90	2789.155	0.276	2789.141	0.288	0.66
1	22	2799.613	0.072	2799.721	0.062	19.92	2799.734	0.063	2799.737	0.065	15.06
2	22	2855.507	0.121	2855.569	0.111	4.28	2855.631	0.124	2855.619	0.120	3.50
0	23	2860.680	0.098	2860.762	0.092	8.15	2860.720	0.101	2860.749	0.099	6.33
3	22	2906.905	0.490	2906.922	0.391	0.44	2906.865	0.435	2906.862	0.445	0.34
1	23	2917.890	0.110	2917.824	0.100	8.31	2917.793	0.097	2917.784	0.101	6.82
2	23	2973.400	0.302	2973.535	0.234	1.80	2973.564	0.235	2973.535	0.217	1.66
0	24	2978.180	0.175	2978.454	0.157	3.09	2978.504	0.151	2978.529	0.145	2.79
3	23	0.000	0.000	0.000	0.000	0.00	3025.061	1.128	3024.682	1.203	0.16
1	24	3035.810	0.174	3036.046	0.166	3.27	3036.058	0.155	3036.048	0.164	2.97
2	24	3092.492	0.577	3092.285	0.433	0.75	3093.036	0.507	3092.795	0.424	0.68
0	25	3097.170	0.419	3096.476	0.403	1.27	3096.850	0.419	3097.107	0.372	1.10
3	24	0.000	0.000	0.000	0.000	0.00	3144.035	1.415	3144.275	1.288	0.07
1	25	3154.703	0.300	3154.229	0.267	1.45	3154.307	0.290	3154.291	0.291	1.21
2	25	3210.654	1.187	3212.063	1.445	0.39	3213.398	1.536	3211.987	0.943	0.30
0	26	3216.451	0.482	3215.846	0.533	0.66	3214.925	1.040	3215.900	0.697	0.48
1	26	3273.587	0.473	3273.266	0.502	0.79	3273.168	0.643	3273.312	0.659	0.62
2	26	3330.030	2.226	3330.323	1.340	0.23	3333.060	2.743	3331.294	1.357	0.20
0	27	3336.009	1.060	3336.187	1.516	0.39	3334.219	1.903	3337.847	1.054	0.33
1	27	3391.761	1.090	3393.623	0.821	0.37	3393.448	0.768	3393.091	0.709	0.39

Table A.3. KIC 8379927: Frequencies and errors (in μHz) for Legacy, Roxburgh (Legacy), Davies, Roxburgh (Davies) and S/N values.

L	n	$\nu_{\rm L}$	$\sigma_{\rm L}$	$\nu_{\rm LR}$	$\sigma_{\rm LR}$	S/N	$\nu_{\rm D}$	$\sigma_{\rm D}$	$\nu_{\rm DR}$	$\sigma_{\rm DR}$	S/N
0	13	0.000	0.000	0.000	0.000	2.25	1728.138	0.435	1728.025	0.141	0.72
1	13	1783.395	0.441	1783.819	0.169	0.54	1783.342	0.273	1783.389	0.164	0.45
0	14	1847.244	0.243	1847.300	0.146	0.76	1847.636	0.854	1847.426	0.174	0.61
1	14	1903.592	0.282	1903.637	0.131	0.68	1904.672	0.846	1903.864	0.191	0.47
2	14	0.000	0.000	0.000	0.000	1.00	1954.857	0.725	1954.580	0.678	0.15
0	15	1967.982	0.255	1967.920	0.154	0.91	1968.190	0.217	1968.206	0.197	0.69
1	15	2023.838	0.297	2023.892	0.230	0.98	2023.666	0.310	2023.654	0.244	0.67
2	15	0.000	0.000	0.000	0.000	1.63	2075.323	0.590	2075.643	0.432	0.26
0	16	2087.937	0.198	2087.844	0.152	1.27	2087.990	0.156	2087.960	0.144	1.05
1	16	2143.133	0.180	2143.140	0.156	1.55	2143.237	0.178	2143.207	0.171	1.14
2	16	2195.244	0.469	2194.891	0.256	0.58	2195.355	0.299	2195.316	0.338	0.43
0	17	2206.506	0.150	2206.692	0.126	1.97	2206.657	0.126	2206.635	0.120	1.78
1	17	2261.245	0.136	2261.106	0.122	2.37	2261.245	0.116	2261.218	0.118	1.88
2	17	2312.707	0.370	2312.251	0.313	0.92	2312.901	0.276	2312.899	0.264	0.74
0	18	2324.439	0.106	2324.451	0.112	3.16	2324.322	0.111	2324.307	0.109	2.80
1	18	2379.779	0.099	2379.770	0.095	3.49	2379.939	0.108	2379.911	0.106	2.72
2	18	2432.197	0.204	2432.318	0.178	1.28	2432.294	0.223	2432.348	0.199	0.95
0	19	2443.152	0.101	2443.116	0.096	4.34	2443.150	0.103	2443.131	0.096	3.61
1	19	2499.437	0.092	2499.391	0.088	5.12	2499.398	0.104	2499.388	0.099	3.65
2	19	2552.415	0.153	2552.284	0.126	1.96	2552.244	0.166	2552.222	0.171	1.35
0	20	2563.543	0.086	2563.596	0.077	6.75	2563.605	0.082	2563.587	0.081	5.00
1	20	2619.926	0.090	2619.986	0.093	6.54	2619.991	0.086	2619.970	0.092	4.81
2	20	2673.136	0.145	2673.038	0.136	2.17	2673.135	0.145	2673.157	0.133	1.69
0	21	2683.948	0.099	2683.962	0.088	6.92	2684.018	0.097	2683.995	0.092	6.02
1	21	2740.437	0.087	2740.463	0.088	6.95	2740.526	0.089	2740.507	0.092	5.25
2	21	2793.385	0.159	2793.453	0.154	2.29	2793.482	0.185	2793.435	0.184	1.70
0	22	2804.566	0.087	2804.494	0.087	7.05	2804.495	0.093	2804.480	0.096	5.06
1	22	2860.993	0.095	2861.027	0.097	6.68	2861.002	0.107	2860.981	0.107	4.89
2	22	2913.836	0.177	2914.001	0.163	2.01	2914.039	0.219	2914.029	0.200	1.56
0	23	2924.530	0.094	2924.483	0.092	6.06	2924.469	0.112	2924.447	0.107	4.42
1	23	2981.323	0.120	2981.304	0.115	5.37	2981.241	0.123	2981.220	0.123	4.06
2	23	3034.225	0.276	3034.302	0.255	1.45	3033.920	0.271	3033.886	0.262	1.15
0	24	3044.841	0.130	3044.850	0.131	3.48	3044.862	0.129	3044.853	0.129	2.92
1	24	3102.033	0.157	3101.964	0.156	3.59	3102.002	0.155	3101.960	0.162	2.80
2	24	3155.043	0.376	3155.020	0.374	0.99	3154.846	0.359	3154.852	0.351	0.77
0	25	3165.482	0.243	3165.537	0.231	1.91	3165.555	0.233	3165.580	0.234	1.60
1	25	3223.052	0.226	3223.183	0.222	2.23	3223.302	0.246	3223.279	0.246	1.74
2	25	3275.789	0.587	3276.510	0.536	0.67	3275.749	0.585	3275.754	0.545	0.53
0	26	3286.718	0.298	3286.792	0.306	1.25	3286.935	0.365	3287.007	0.329	0.97
1	26	3344.137	0.294	3344.679	0.287	1.50	3344.479	0.386	3344.531	0.363	1.11
2	26	3397.777	0.684	3398.078	0.608	0.46	3397.302	0.668	3397.091	0.721	0.34
0	27	3408.187	0.477	3409.214	0.433	0.81	3408.663	0.512	3408.606	0.532	0.61
1	27	3465.693	0.433	3466.067	0.419	0.93	3466.404	0.472	3466.430	0.499	0.69
2	27	3518.572	0.951	3518.691	1.077	0.26	3520.401	1.085	3519.568	1.151	0.21
0	28	3531.333	0.741	3531.243	0.775	0.41	3531.755	0.927	3532.584	1.107	0.34
1	28	3587.270	0.657	3587.792	0.810	0.53	3587.318	1.097	3587.141	1.312	0.39
2	28	3640.416	1.853	3641.615	1.818	0.17	3641.946	1.253	3643.727	2.903	0.12
0	29	3651.161	0.840	3650.143	1.027	0.28	3650.679	0.795	3649.769	1.716	0.19
1	29	3710.839	0.840	3710.035	0.897	0.36	3710.678	0.938	3710.382	1.182	0.27
2	29	3762.282	1.692	3771.014	7.568	0.12	3762.746	1.892	3764.758	2.095	0.10
0	30	3769.722	1.096	3768.325	3.623	0.19	3770.309	1.537	3769.174	1.213	0.19
1	30	3836.226	1.273	3837.493	1.676	0.20	3835.787	1.141	3836.531	0.888	0.24