

# First *Kepler* results on compact pulsators VIII: Mode identifications via period spacings in $g$ –mode pulsating Subdwarf B stars

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## ABSTRACT

We investigate the possibility of nearly-equally spaced periods in 13 hot subdwarf B (sdB) stars observed with the *Kepler* spacecraft and one observed with CoRoT. Asymptotic limits for gravity ( $g$ –)mode pulsations provide relationships between equal period spacings of modes with differing degrees  $\ell$  and relationships between periods of the same radial order  $n$  but differing degrees  $\ell$ . Period transforms, Kolmogorov-Smirnov tests, and linear least-squares fits have been used to detect and determine the

significance of equal period spacings. We have also used Monte Carlo simulations to estimate the likelihood that the detected spacings could be produced randomly.

Period transforms for nine of the *Kepler* stars indicate  $\ell = 1$  period spacings, with five also showing peaks for  $\ell = 2$  modes. 12 stars indicate  $\ell = 1$  modes using the Kolmogorov-Smirnov test while another shows solely  $\ell = 2$  modes. Monte Carlo results indicate that equal period spacings are significant in 10 stars above 99% confidence and 13 of the 14 are above 94% confidence. For 12 stars, the various methods find consistent regular period spacing values to within the errors, two others show some inconsistencies, likely caused by binarity, and the last has significant detections but the mode assignment disagrees between methods.

We use asymptotic period spacing relationships to associate observed periods of variability with pulsation modes for  $\ell = 1$  and 2. From the *Kepler* first year survey sample of 13 multiperiodic  $g$ -mode pulsators, five stars have several consecutive overtones making period spacings easy to detect, six others have fewer consecutive overtones but period spacings are readily detected, and two stars show marginal indications of equal period spacings. We also examine a  $g$ -mode sdB pulsator observed by CoRoT with a rich pulsation spectrum and our tests detect regular period spacings.

We use Monte Carlo simulations to estimate the significance of the detections in individual stars. From the simulations it is determined that regular period spacings in 10 of the 14 stars is very unlikely to be random, another two are moderately unlikely to be random and two are mostly unconstrained.

We find a common  $\ell = 1$  period spacing spanning a range from 231 to 272 s allowing us to correlate pulsation modes with 222 periodicities and that the  $\ell = 2$  period spacings are related to the  $\ell = 1$  spacings by the asymptotic relationship  $1/\sqrt{3}$ . We briefly discuss the impact of equal period spacings which indicate low-degree modes with a lack of significant mode trappings.

**Key words:**

Stars: oscillations – Stars: subdwarfs

## 1 INTRODUCTION

Asteroseismology is the process in which stellar pulsations are used to discern the physical condition of stars. The process includes matching stellar models to observations, associating periodicities with pulsation modes, and examining where the models succeed and fail. Slight mismatches between observations and models can provide insights to new physics or add constraints

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to previously assumed conditions. Examples include using deviations from sinusoidal variations to constrain convective depths (Handler et al. 2008; Montgomery 2005) and using deviations from equally spaced overtones to discern interior composition gradients (??, and references therein). In many cases, the best results are achieved for stars with highly constrained observations. Observational constraints can include the usual spectroscopic measurements ( $\log g$ ,  $T_{\text{eff}}$  and some compositional constraints), number and characterization of periodicities (frequency or period, amplitude, phase, and pulse shape) as well as frequency multiplets and equal period spacings which associate specific periodicities with pulsation modes.

In brief, nonradial pulsations (periodicities) are characterized by three quantized numbers (modes)  $n$ ,  $\ell$ , and  $m$ . These represent the number of radial nodes ( $n$ ), surface nodes ( $\ell$ ) and azimuthal surface nodes ( $m$ ). In the asymptotic limit for  $n \gg \ell$ , gravity ( $g$ -)modes should be equally spaced in period for consecutive values of  $n$  according to the expression:

$$\Pi_{\ell,n} = \frac{\Pi_o}{\sqrt{\ell(\ell+1)}}n + \epsilon \quad (1)$$

where  $\Pi_o$  and  $\epsilon$  are constants, in seconds (Aerts, Christensen-Dalsgaard, Kurtz 2010; Unno et al. 1979). The period spacings between two consecutive overtones are:

$$\Delta\Pi_{\ell} = \frac{\Pi_o}{\sqrt{\ell(\ell+1)}} \quad (2)$$

where  $\Delta\Pi_{\ell} = \Pi_{\ell,n+1} - \Pi_{\ell,n}$ . Because of geometric cancellation (Reed et al. 2005),  $\ell = 1$  and 2 modes are the most likely nonradial modes to be observed and the specific relations between them are:

$$\Pi_{n,\ell=2} = \frac{\Pi_{n,\ell=1}}{\sqrt{3}} + C \quad (3)$$

where  $C$  is a constant that is expected to be small and is zero if  $\epsilon_2 = \epsilon_1$ , and

$$\Delta\Pi_{\ell=2} = \frac{\Delta\Pi_{\ell=1}}{\sqrt{3}}. \quad (4)$$

The asymptotic approximation applies to periods within completely homogeneous stars. However real stars, particularly compact stars for which gravitational settling is important and hot stars in which radiative levitation is important, develop compositional discontinuities where the mean molecular weight changes. The *transition zones* of compositional changes can work as a reflective wall which confines pulsations to specific stellar regions. This “trapping” of pulsation modes changes the spacing between consecutive overtones compared to the average spacing  $\Delta\Pi$ . Trapped modes can be used to deduce structural changes associated with chemical transitions (e.g. Kawaler & Bradley 1994; Costa et al. 2008).

The period spacing relations are independent of  $m$  and are applied under the assumption of  $m = 0$  periodicities. In the case of extremely slowly rotating stars, this may be a valid assumption. However for stars which complete several revolutions within a set of observations, pulsations will create frequency multiplets. These multiplets will have  $2\ell + 1$  components spaced at

$$\nu_{n,\ell,m} = \nu_{n,\ell,0} + m\Omega(1 - C_{n,\ell})$$

where  $\Omega$  is the rotation frequency and  $C_{n,\ell}$  is the Ledoux constant (Ledoux 1951).

In this paper, we apply Eqns. 1 to 4 to  $g$ -mode pulsations observed in hot subdwarf (sdB) variables. Subdwarf B variables were first discovered in 1996 and now consist of two well-established classes. These are the short-period pressure ( $p$ -)mode pulsators which are designated V361 Hya stars (Kilkenny et al. 1997) and longer-period gravity ( $g$ -)mode pulsators designated V1093 Her stars (Green et al. 2003). There are also hybrid pulsators, sometimes called DW Lyn stars after that prototype (Schuh et al. 2006), which show both types of variations. About 50 V361 Hya pulsators have been detected (Østensen et al. 2010a) with a couple dozen receiving various amounts of follow-up data (see for example Reed et al. 2007). However, observational constraints on pulsation modes are extremely rare for the V361 Hya class, occurring only twice using multiplets Reed et al. (2004); Baran et al. (2009). Time-resolved spectroscopy, sometimes coupled with multicolor photometry, has had some limited success (Telting & Østensen 2004, 2006; Reed et al. 2009; Baran et al. 2010b). See §2.2 through 2.4 of Østensen (2010) for a recent review of these methods. The lack of observational constraints has led model-matching efforts to proceed by using the forward method, which consists of matching observed periods to those of models, with the closest fit, within spectroscopic constraints, being deemed the correct one (For a review see Charpinet et al. 2009). Progress on  $g$ -mode pulsators has been slow because of the difficulties in observing many pulsation cycles for periodicities of one to three hours in extremely blue stars from the ground. With the acquisition of long time-series photometric data from satellites, such as *Kepler* and CoRoT, detailed asteroseismology of the V1093 Her pulsators is now possible.

The *Kepler* spacecraft has a primary mission to find Earth-sized planets within the habitability zone around Sun-like stars (Borucki et al. 2010). To do this, the spacecraft continuously examines roughly 150,000 stars in search of transits. As a byproduct of that search, high quality photometric observations are obtained which have proven extremely useful for the study of variable stars (Koch et al. 2010; Prsa et al. 2010). The *Kepler* spacecraft has two effective integration times: A short cadence (SC) integration near 1 minute and a long cadence (LC) integration near 30 minutes. The first year of the *Kepler* mission was dedicated to a survey phase where many target buffers were

**Table 1.** Properties of the stars in this paper. Columns 1 and 2 supply the Kepler Input Catalog number and a more common name, columns 3 and 4 provide the *Kepler* magnitudes ( $K_p$ ) and estimated contamination factors (Q), columns 5 and 6 supply spectroscopic parameters, column 7 lists if the star is in a known binary (RE = reflection effect binary; EB = eclipsing binary; EV = ellipsoidal variable binary; and the inferred companion is in parentheses), and column 8 lists references from this series of papers. RHØ indicates Østensen et al. (2010c) and CH10 indicates Charpinet et al. (2010).

KIC	Name	Q	$K_p$	$F_{cont}$	$T_{eff}$	$\log g$	Binary	Ref
2697388	J19091+3756	2.3	15.39	0.149	23.9(3)	5.32(3)	-	I,III
2991403	J19272+3808	1	17.14	0.601	27.3(2)	5.43(3)	RE(dM)	I,V
3527751	J19036+3836	2.3	14.86	0.081	27.9(2)	5.37(9)	-	I,III
5807616	KPD 1943+4058	2.3	15.02	0.332	27.1(2)	5.51(2)	-	I,III,IV
7664467	J18561+4319	2.3	16.45	0.879	26.8(5)	5.17(8)	-	I,III
7668647	FBS1903+432	3.1	15.40	0.226	27.7(3)	5.45(4)	-	VI,VII
8302197	J19310+4413	3.3	16.43	0.256	26.4(3)	5.32(4)	-	VI,VII
9472174	2M1938+4603	0	12.26	0.022	29.6(1)	5.42(1)	EB(dM)	I,RHØ
10001893	J19095+4659	3.2	15.85	0.710	26.7(3)	5.30(4)	-	VI,VII
10553698	J19531+4743	4.1	15.13	0.385	27.6(4)	5.33(5)	-	VI,VII
10670103	J19346+4758	2.3	16.53	0.450	20.9(3)	5.11(4)	EV(WD)	I,III
11179657	J19023+4850	2.3	17.06	0.129	26.0(8)	5.14(13)	RE(dM)	I,V
11558725	J19265+4930	3.3	14.95	0.028	27.4(2)	5.37(3)	-	VI,VII
-	KPD0629-0016	-	14.91 <sup>†</sup>	-	27.8(3)	5.53(4)	-	CH10

assigned to SC observations, which switched targets on a monthly basis (Jenkins et al. 2010). Papers I through VII (Østensen et al. 2010b,d; Kawaler et al. 2010a,b; van Grootel et al. 2010; Reed et al. 2010; Baran et al. 2010a) of this series along with Østensen et al. (2010c) describe the search and resulting detections of periodicities in compact stars from *Kepler* survey phase observations. Papers I and VI provide a spectroscopic analysis and pulsation overview of all compact stars observed with *Kepler*; Papers II, III, V, VII, and Østensen et al. (2010c) describe the specific pulsation periods of the pulsators and Paper IV associates a model with one pulsator using the Forward Method. Those papers serve as a complete introduction to pulsating sdB stars using *Kepler* SC observations. Additionally, CoRoT (COnvection, ROtation, and planetary Transits satellite)<sup>1</sup> has observed one V1093 Her pulsator; KPD 0629-0016 (hereafter KPD 0629; Charpinet et al. 2010).

## 2 DETECTION AND SIGNIFICANCE OF REGULAR PERIOD SPACINGS

In Paper III (Reed et al. 2010) we introduced the possibility that V1093 Her stars could show evenly spaced periods as an indicator of pulsation degree  $\ell$ . Now we expand that search to all V1093 Her stars with space-based (13 *Kepler* and 1 CoRoT) observations. Basic information for the 14 stars of this study are provided in Table 1. This includes Kepler Input Catalog (KIC) numbers, stellar designations from other sources, and spectroscopic properties from Papers I and VI (except for KPD 0629, which are from Charpinet et al. 2010).

<sup>1</sup> <http://smc.cnes.fr/COROT/>

From our work with KIC10670103 (Paper III), we were expecting  $\ell = 1$  period spacings near to 250 s and there are several other stars (particularly KIC8302197 and KIC10001893) which trivially show equal period spacings (or multiples thereof) very near to this value. Since V1093 Her stars have small ranges for  $T_{\text{eff}}$  and  $\log g$ , we anticipated that all our targets should have  $\ell = 1$  period spacings near to 250 s. We took the dual approach of Winget et al. (1991), to search for regular period spacings using period transforms (PT) and Kolmogorov-Smirnov (KS) tests. The period transform is an unbiased test where power spectra are converted to period spectra and then a Fourier transform is taken of that. Peaks in the PT indicate common period spacings. We used the  $g$ -mode region from 0 – 1000  $\mu\text{Hz}$  from our power spectra. The PT method is sensitive to the number of periods from which to find correlations. Winget et al. (1991) did this for the pulsating DOV star PG 1159-035 (also known as GW Vir), for which 125 periods were detected. Conversely, our richest  $g$ -mode pulsator only has 46 periodicities and our poorest a meagre seven. As such, our expectations were low and we were happily surprised by the success of this method. For 10 stars (shown in the left panels of Fig. 1) the  $\ell = 1$  peak corresponding to a regular period spacing near 250 s is readily picked out and in five of these, we can deduce the  $\ell = 2$  peak as well using Eqn. 4. We then fitted the PT with a nonlinear least-squares technique to determine the period spacing values and errors for each one. For KIC3527751, an alias occurs for  $\Delta\Pi_1 + \Delta\Pi_2$  and KIC11558725's peaks are split because of small period spacings (possibly related to rotational multiplets). KIC8302197 and KIC9472174 do not have any peaks that stand out. For KIC8302197, this most likely occurs because of the few periods (9) and for KIC9472174 this is likely related to the short data series (9.7 d) and the complexity within the FT caused by binarity.

The Kolmogorov-Smirnov (KS) test is a nonparametric test that compares a sample distribution ( $F_n(x)$ ) with a reference distribution (Eqn. 1 in our case Chakravarti, Laha, Roy 1967). The KS test has proven useful with white dwarf pulsators (Winget et al. 1991; Kawaler 1988). The KS test uses previously detected pulsation periods as input and so has a selection effect caused by our detections. Any such effect should be small as the data are nearly gap-free, and so period detections should be accurate. However, some stars show small, marginally-unresolved periodicities and these could skew the results as they are sometimes included and other times excluded in the period lists. We applied the KS test for equal period spacings between 50 and 800 seconds. The results for the range of 100 – 300 s are shown in the right panels of Fig. 1. Unlike the PT test, the KS test has a local minimum for all stars for period spacings near 250 s, except for KIC3527751, where it detects spacings of 152 s. The PT test for KIC3527751 shows both the  $\ell = 1$  and 2 peaks, with the  $\ell = 1$  having a higher amplitude, whereas the KS test only detects an extremely strong  $\ell = 2$

spacing. The  $\ell = 2$  spacing is within the errors of the both the PT test and a linear least-squares fit and match the value determined using Eqn 4 based on our  $\ell = 1$  determinations. However, only for the KS test does it dominate. The  $\Delta\Pi = 241$  s detection for KIC9472174 is not significant as there are many small spacings between 100 and 150 s, likely due to the binarity. While the PT test was not useful for KIC8302197 as it has too few periods, the KS test readily found a regular period spacing. All of the KS results show some quantity of multiple peaks caused by deviations in the period spacings. In PG 1159, these are attributed to mode trapping. Stars which show weak secondary peaks indicative of  $\ell = 2$  spacings include KICs 2991403, 5807616, 7664467, 7668647, 10553698, 11558725, and KPD 0629. Surprisingly, the  $\ell = 2$  spacings for KIC10670103 produce an insignificant peak, even though we previously detected eight  $\ell = 2$  modes including five consecutive overtones.

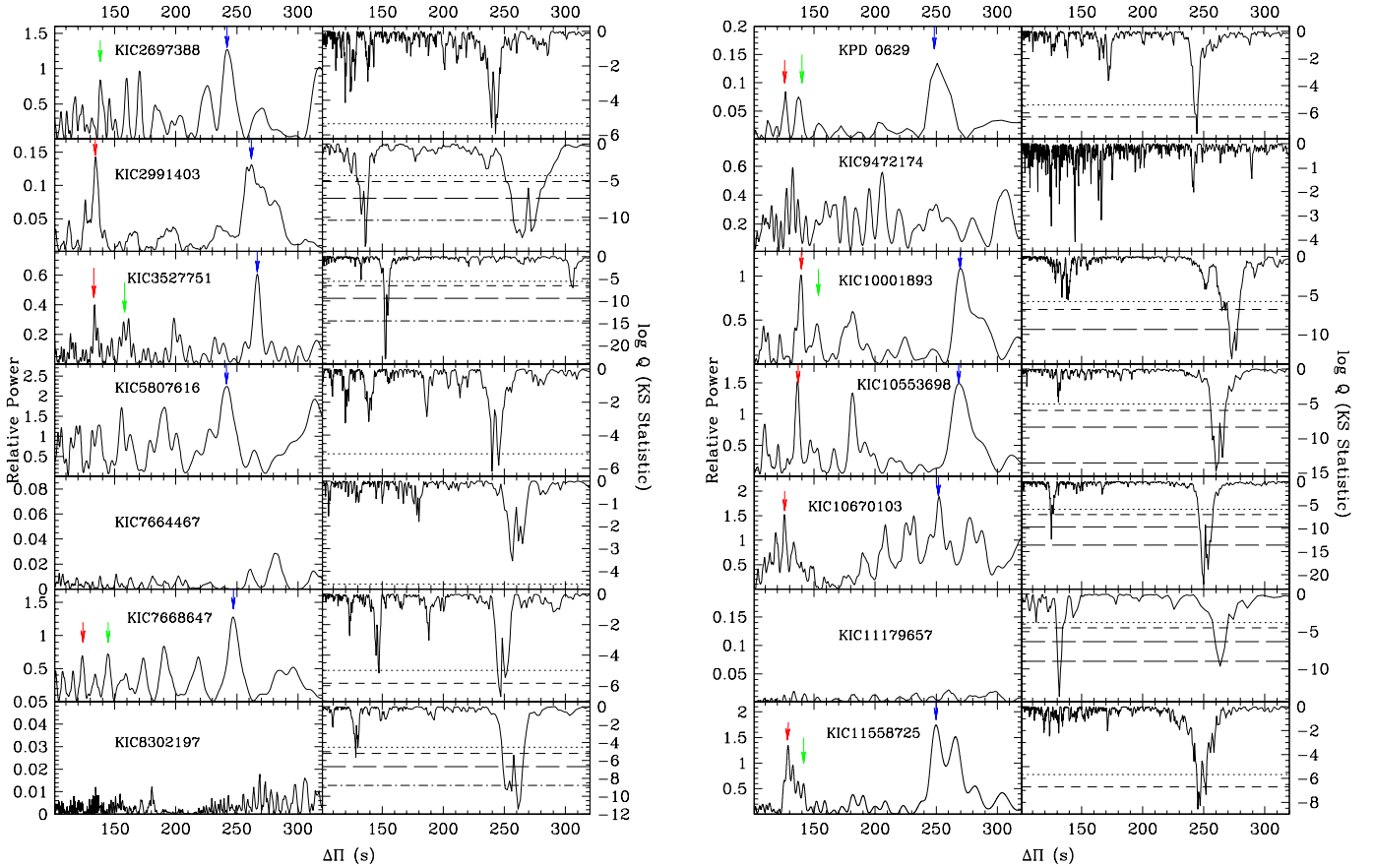
Using the period spacings found in the PT and KS tests (or integer multiples thereof), we identified periods as  $\ell = 1$  or 2, or unknown<sup>2</sup>. We then did a least-squares straight line fit to each  $\ell = 1$  or 2 series, arbitrarily assigning  $n$  values such that  $n$  was not negative<sup>3</sup> and satisfying Eqn. 3 between the  $\ell = 1$  and 2 series. The period spacings found using all three methods were in agreement and in Table 2 we use those from the linear least-squares fits, for which the errors are the most straightforward.

## 2.1 Monte Carlo tests

From the PT and KS tests, we already have strong evidence that nearly all of these stars have regular period spacings. However, according to stellar models (Charpinet et al. 2002, hereafter CH02), even period spacings are not anticipated. Since most of the pulsators have rich pulsation spectra, it is reasonable to question if the detections are just chance alignments. As a third test, we produced Monte Carlo simulations that randomly select periods to match with asymptotic sequences to within the errors. A number of observed periodicities,  $N$ , set to match what is observed, were randomly selected to fit within an observed range  $P_{\min} \leq P \leq P_{\max}$ . For KIC8302197 and KIC7664467, we compared the  $N$  randomly selected periods with a single sequence of the form  $P_n = (P_{\min} - j\delta) + n \times \Delta\Pi \pm \sigma$ . The quantity  $\delta$  represents a small shift of the zero point which was repeated  $j$  times until  $P_0 \leq P_{\min} - \Delta\Pi$ . The  $j$  value that produced the greatest number of matches was used and the  $n$  values were tested to see how many consecutive overtones were detected.  $\sigma$  is

<sup>2</sup> Periods and mode identifications appear in Tables 4 through 17 of the on-line appendix.

<sup>3</sup> Except for KIC5807616, where  $n$  is chosen to match Paper IV.



**Figure 1.** Left panels: Indications of regular period spacings using Period transforms. Blue arrows indicate the  $\ell = 1$  spacings, green arrows the  $\ell = 2$  spacings, and red arrows aliases. Right panels: The Kolmogorov-Smirnov (KS) test applied to the detected periodicities. Confidence levels of 90% (dotted line), 95% (short dashed line), 99% (long dashed line), and 99.9% (dot-dashed line) are shown. These confidence levels are calculated for the range of period spacings between 100 and 400 seconds, assuming uniformly distributed random periods.

the difference allowed between the random and sequence periods and is chosen to be slightly bigger than what is observed. Choosing this value for  $\sigma$  makes our Monte Carlo simulations extremely conservative and the real probability of matching is much smaller since this  $\sigma$  is typically valid for only one or two periods, with most errors being much smaller.  $n$  is then stepped until  $P_n > P_{\max}$ . To generate an  $\ell = 2$  appropriate sequence, we simply divided the  $\ell = 1$   $P_n$  by  $\sqrt{3}$ , as required by Eqn. 3 and then extended  $P_{\max}$  so even the longest random periods could have  $\ell = 2$  matches. Our code also insured that only one randomly selected period matched each possible sequence period (eliminating double counting). A million sets of random periods were generated in each Monte Carlo simulation and the resulting matches were converted to percentages in Column 9 of Table 2.

The Monte Carlo simulations indicate that 10 stars have less than a 1% chance that their regular period spacings are the product of random chance. KPD 0629 has a 1.1% chance and



**Table 2.** Period spacings determined from linear least-squares fits. Column 1 provides the KIC number (KPD designation for the CoRoT star), Columns 2 and 3 are the  $\ell = 1$  and 2 period spacings (errors in parentheses), columns 4, 5, 6, 7, and 8 provide the total number of periods, the number assigned as  $\ell = 1$  and 2, and the number of consecutive  $\ell = 1$  and 2 overtones. Parenthetic numbers in Column 6 indicate the number of modes which are ambiguous between  $\ell = 1$  and 2 identifications. They are not counted as  $\ell = 2$  in Column 6. The last column provides the percentage of Monte Carlo simulations that produced a match to the observations. Notes: a) Using just the  $\ell = 1$  sequence with five consecutive overtones. b) Leaving three deviant  $\ell = 2$  matches as unassigned. c) 10 million simulations produced no results which included 13 consecutive overtones. d) Assuming f1 is  $\ell = 1$ . e) Periods f1 and f2 are counted as  $\ell = 2$ . f) Leaving the deviant periods f17 and f44 unassigned and assuming f2 is  $\ell = 2$ .

Star	$\Delta\Pi_1$	$\Delta\Pi_2$	N	$N_1$	$N_2$	$NC_1$	$NC_2$	MC%
2697388	240.07 (0.27)	138.54 (0.16)	36	16	13 (2)	4	3	0.04
2991403	268.52 (0.74)	153.84 (1.19)	16	7	4	5	0	0.009 <sup>a</sup>
3527751	266.10 (0.38)	153.57 (0.12)	38	15	14 (5)	2	2	0.018
5807616	242.12 (0.62)	139.13 (0.38)	22	11	6 (3)	3	3	23.0
7664467	260.02 (0.77)	-	7	6	1	0	-	0.16
7668647	248.15 (0.44)	144.71 (0.57)	18	12	5 (2)	2	0	0.0014
8302197	257.70 (0.56)	-	9	9	-	2	-	0.0007
9472174	255.63 (0.30)	147.70 (0.69)	20	8	8 (1)	2	2	5.4 <sup>b</sup>
10001893	268.53 (0.61)	154.74 (0.34)	26	18	9	12	3	0.0 <sup>c</sup>
10553698	271.15 (0.54)	156.68 (0.31)	30	12	9 (6)	6	3	0.22
10670103	251.13 (0.31)	145.59 (0.26)	27	19	8	5	5	0.04
11179657	231.02 (0.02)	133.64 (0.40)	12	3	7 (1)	0	3	0.0002 <sup>d</sup>
11558725	246.77 (0.58)	142.57 (0.14)	46	18 <sup>e</sup>	13 (1)	8	2	2.2 <sup>f</sup>
KPD0629	247.17 (0.48)	142.74 (0.30)	17	12	3 (2)	3	2	1.1

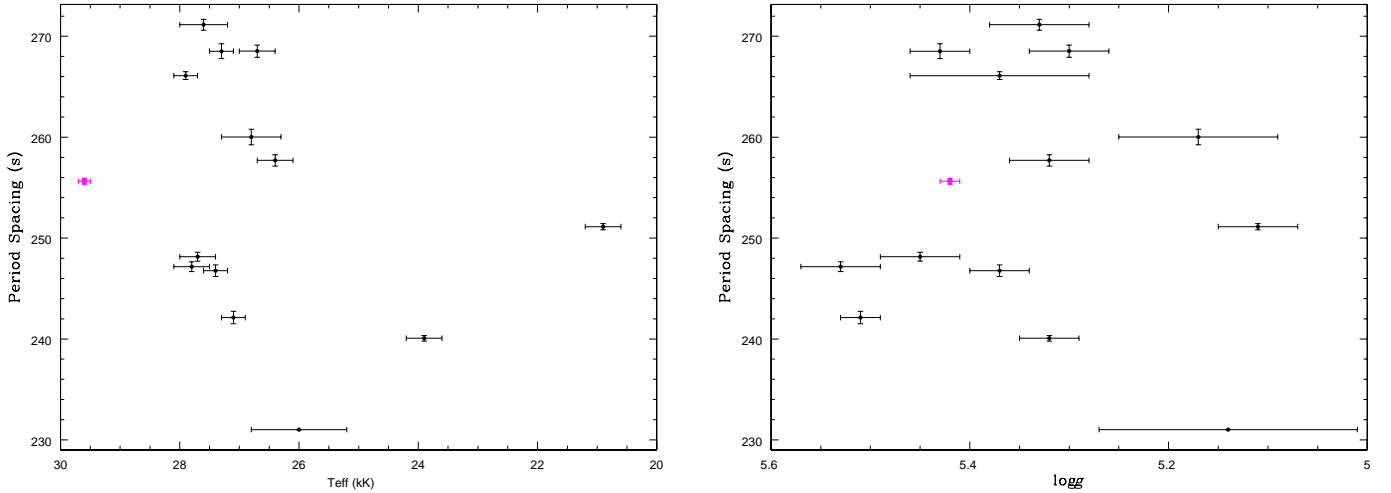
**Table 3.** Sample table of supplemental material. Tables for all 14 stars appear on-line. Periods and period spacings for KIC2697388. Identifications (column 1) and periods are those from Paper III (Reed et al. 2010). Columns 3, 4, and 5 provide the mode degree  $\ell$  and the overtone fit to  $P_\ell = P_{\ell o} + n \cdot \Delta P_\ell$  where n is arbitrarily chosen such that there are no negative values, except for KIC5807616, where it is chosen to match Paper IV. Column 6 provides the difference between the observed and asymptotic relation period, column 7 lists the fractional period differences and column 8 is the observed spacing  $P(n_{\ell,i}) - P(n_{\ell,j})/(i - j)$ . It is ambiguous whether  $\ell = 1$  or 2 modes should be associated with f25, f23, and f11. f25 was not used for the  $\ell = 1$  fit as it is most likely  $\ell = 2$ .

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f30	2757.118	2	-	15	19.393	0.140	140.7
f29	3008.732	2	-	17	-6.072	0.044	125.8
f28	3517.219	1	10	-	-8.790	-0.037	254.2
f27	3700.224	2	-	22	-7.277	-0.053	138.3
f26	3757.261	1	11	-	-8.815	-0.037	240.0
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

KIC11558725 has a 2.2% chance of occurring randomly from our Monte Carlo simulations. KIC5807616 and KIC9472174 are unconstrained from this test.

### 3 ENSEMBLE AND MODEL COMPARISON

Figure 2 shows our detected  $\ell = 1$  period spacings with gravity and effective temperature. The temperatures span nearly 10 000 K while  $\log g$  only covers 0.6 dex. Naturally, what is sought is a relationship between period spacings and physical properties, such as Eqn. 5 of Kawaler & Bradley (1994) for white dwarfs. Such a relationship would allow the determination of properties based on period spacings alone. While white dwarfs and sdB stars are both compact stars, there is no *a priori* reason to expect that any correlations should exist for sdB stars. Figure 31 of CH02 indicates

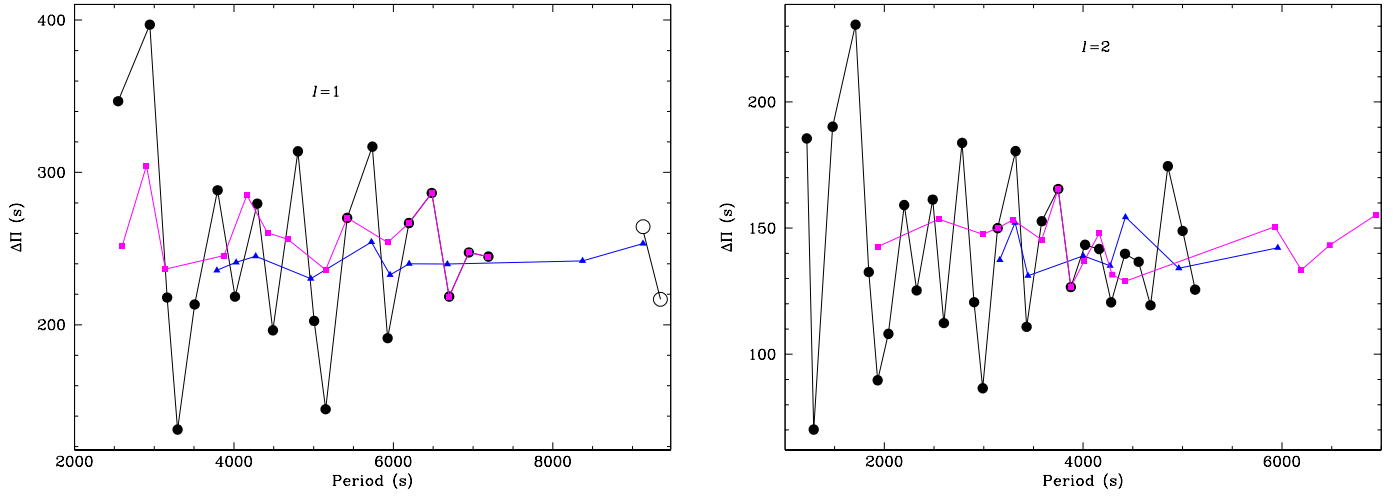


**Figure 2.** Period spacings compared with  $T_{\text{eff}}$  and  $\log g$ . The red point indicates KIC9472174, which is the only star for which PT, KS, and MC tests were all inconclusive.

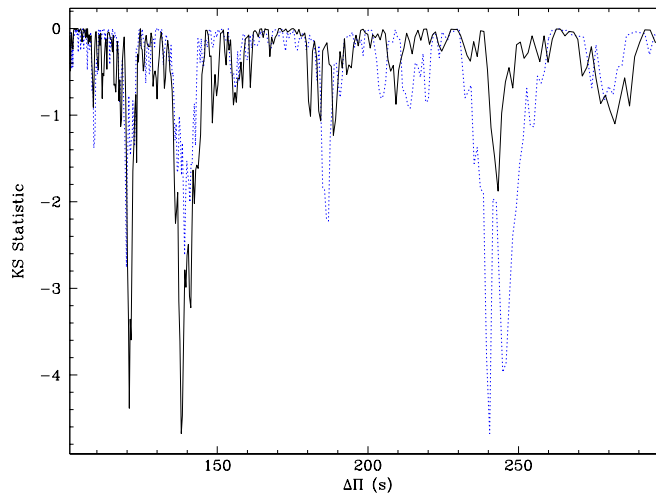
that as envelope thickness decreases, the distance between trapped modes increases as do period spacings. However, the effect of trapped modes is increased with decreasing envelope thickness, and so while there are more overtones between trapped modes, the impact of a trapped mode would be to eliminate any sequence of the form of Eqn. 1 longer than three or four consecutive periods. Figure 16 of CH02 indicates that the longest period spacings should occur where  $T_{\text{eff}}$  and  $\log g$  are both small or both large, though they only test for  $g$ -modes with  $n \leq 9$ , which may be too small for asymptotic relations. However, in Fig 2 there do not appear to be any trends, either with gravity or temperature. Since 10 stars have temperatures near 27 500 K yet period spacings that range from 242 to 271 s while the extremophiles of the group have period spacings near the middle of this range, it would have to be deduced that temperature does not impact period spacings in sdB stars. No trends are obvious with  $\log g$  either, though in this case the span is much smaller compared with the associated errors. Table 3 of CH02 indicates that period spacing should increase with decreasing envelope mass. It would be useful to compare the CH02 models with Paper IV, but unfortunately the CH02 paper calculates for  $\ell = 3$  modes and Paper IV does not, making a direct comparison difficult. Appropriate stellar models will have to be produced to determine what the parameter(s) is (are) that affects the period spacings, but this paper is concerned with interpreting observations and so we will not address modeling issues.

While CH02 examined period spacings for gravity (and pressure) modes, the model they used was significantly hotter than these stars. The model of Paper IV is obviously appropriate as it was made to match KIC5807616 and so we compared it with our findings. Figure 3 shows the model spacings for many of the  $\ell = 1$  and 2 modes (black circles). The  $\ell = 1$  period spacings

range from  $\approx 50$  to 400 s with mode trapping dominating the spacings. In the sequence of 21 period spacings, only twice is the change between consecutive spacings smaller than 20 s while the rest are greater than 50 s. For comparison, the period spacings we selected for KIC5807616 (which changed by less than 25 s for all  $\ell = 1$  modes) are shown as blue triangles. Naturally, one could pick out just the peaks or troughs of the model and get more consistent period spacings that way, but you would only rarely get a sequence of three consecutive overtones. To test this assumption we performed a blind test on 51 model  $\ell = 1$  and 2 periods from Paper IV; including model sequences of 21 consecutive  $\ell = 1$  and 30  $\ell = 2$  modes. Putting them in period order only (removing the model mode assignments) and using the observed period spacings as a guide, we assigned periods as  $\ell = 1$  or 2, or left them unassigned. Allowing periods to deviate by up to 32 s from equal spacings (28% more than the observed deviations), we assigned 15  $\ell = 1$  and 16  $\ell = 2$  modes (double counting eight periods, which were ambiguous between the modes). Of the 15  $\ell = 1$  assignments, eight were model  $\ell = 1$  modes and of the 16 possible  $\ell = 2$  mode assignments, eight were model  $\ell = 2$  modes (two others were close). Our mode assignments from the model periods are shown as (magenta) squares in Fig. 3. When squares are plotted over circles, our blind test mode assignments match those of the model. As expected, this test indicates that mode identifications using equal period spacings does not work well if there is any significant mode trapping since Eqn. 1 biases us to selecting periods with small (or no) mode trapping. We also applied the KS test to the 51 model periods and the results are shown in Fig. 4. The KS test preferentially detects  $\ell = 2$  period spacings with a mild  $\ell = 1$  period spacing. For comparison, the KS test for KIC5807616's observed periods is shown as a dotted (blue) line and shows that the actual data has much stronger  $\ell = 1$  period spacings. However, the period spacings detected in the models are about correct, indicating that perhaps with more subtle mode trapping, the model would better approximate the observations. In Table 6 of the supplemental material, we show our mode assignments as well as those of Paper IV. We chose the radial order  $n$  to match the model at  $f_{11}=4027$  s. When mode assignments via regular period spacings and those from the model agreed, so did the radial order. Four of our 11  $\ell = 1$  mode assignments matched those of the model and seven of our nine  $\ell = 2$  mode assignments matched. Again, this likely indicates that the star does not trap modes as significantly as the model predicts. Since this paper is concerned with observed mode identifications and period spacings, we leave a detailed model analysis to those best suited to do them.



**Figure 3.** A comparison between model, observed, and linearly fit model data for KIC5807616. Black circles are model periods from Paper IV (open circles were not used in the fit), blue triangles are those found from *Kepler* data and magenta squares are those found from a blind fit using the observed period spacing with model periods. If a magenta square is plotted over a black circle, then our blind fit matched the model's mode assignments.



**Figure 4.** The KS test applied to 51 model periods from Paper IV. The dotted (blue) line is the KS test for KIC5807616 from Fig. 1.

## 4 SUMMARY

We tested 13 *Kepler*-observed and one CoRoT-observed  $g$ -mode pulsating subdwarf B stars for consistent period spacings which can be used to observationally identify pulsation modes. We used two different spacings detection tests, a period transform (PT) and a Kolmogorov-Smirnov (KS) test and a Monte Carlo (MC) significance test. The PT test identified 10 stars as having consistent  $\ell = 1$  period spacings and five of these also showed indications of  $\ell = 2$  period spacings. Our KS test results clearly detected  $\ell = 1$  period spacings in *all* our program stars, except KIC9472174, which has a period spectrum complicated by binarity (though the KS result does have an appropriate local minimum) and KIC3527751, for which it finds a very strong  $\ell = 2$  period spacing.

A further five stars show local minima appropriate for  $\ell = 2$  period spacings from their KS tests. Monte Carlo tests indicate that for 10 stars, our mode assignments (provided in accompanying on-line material) are very likely correct. For three additional stars, a random cause for the spacings is below 6% and KIC5807616 has a 23% chance that the equal period spacings are being created randomly, as a worst-case scenario.

For all sample stars, except KIC9472174, three of the four methods (PT, KS, MC, and linear least-squares) find evidence for regular period spacings. 12 of the 14 stars have  $\ell = 1$  and 2 period spacings which satisfy Eqn. 4 and 11 stars have periods (45 periods in total) that satisfy Eqn. 3, with  $C$  equal to zero. Combined, these provide a strong indicator that we are correctly identifying periodicities as  $\ell = 1$  and 2 modes, rather than higher degrees which have different relations. For these 14 stars, we assigned a total of 222 of a possible 317 periodicities as  $\ell = 1$  or 2 modes. Such a large quantity of observationally constrained modes should prove exceedingly useful for stellar modeling.

Our results clearly show the value of long-duration space-based observations. While there have been some remarkable ground-based efforts to observe  $g$ -mode sdB stars, they have not resulted in sufficient detections to evaluate period spacings. Additionally, the *Kepler* results are solely from the survey phase of the mission. Longer duration observations should detect more pulsation periods, including higher degree ( $\ell \geq 2$ ) modes, which we have not searched for at all.

## 5 CONCLUSION

In order for Eqn 1 to be useful, mode trapping must be small (or none). Since Eqn 1 produced a large fraction of significant mode assignments for nearly all of the stars we examined, mode trapping must be substantially reduced from what current models indicate. Figure 16 of CH02 shows period spacings against both  $T_{\text{eff}}$  and  $\log g$  for  $g$ -mode pulsations. Unfortunately, it only has  $n \leq 9$ , where evenly-spaced periods are not expected. However, for higher  $n$  values, such a plot should show a flat surface. According to CH02,  $\Delta\Pi$  shows a plateau of maximum values running from the lowest  $T_{\text{eff}}$  and  $\log g$  to the highest  $T_{\text{eff}}$  and  $\log g$ . However those results are significantly affected by mode trapping, and so may not be a clear indicator of trends in period spacings. Minimal mode trapping could be an indicator that sdB stars are not as chemically stratified as models usually presume. Perhaps thermohaline convection, caused by an inverse  $\mu$ -gradient, as described by Théado et al. (2009) could increase mixing and reduce chemical stratification.

Our assignments as  $\ell = 1$  or 2 modes also adds constraints to driving models. While Paper IV

produced models with driven modes in the correct period range, previous modeling work (including Hu et al. 2009; Jeffery and Saio 2007; Fontaine et al. 2006, among others) had difficulties. Those models preferentially found  $\ell \geq 4$  to be driven (also at temperatures cooler than observed). This is contradicted by our results, which clearly follow Eqns. 3 and 4, indicating  $\ell = 1$  and 2 modes.

Prior to space-based data such as *Kepler* and CoRoT, it seemed unlikely that sdB asteroseismology using  $g$ -modes to probe the core would bear fruit. The discovery of equal period spacings will now have changed that as we can readily correlate modes with periodicities. The forward method of mode assignment is no longer necessary for these stars, which now provide a new modeling challenge. That challenge will be to model stars like KIC10670103, KIC10001893, and KIC10553698 which have lengthy sequences of successive overtones, equal period spacings which show minimal indications of mode trapping, and provide tens of periods with secure mode assignments each.

We anticipate that once longer-duration *Kepler* data are available, many more pulsation periods will be detected. Already there are typically too many periods to be accounted for solely using  $\ell = 1$  and 2 modes and that problem will be compounded. It is anticipated that the extra periodicities will be accounted for using higher degree modes. Such an event will require more sophisticated techniques and tests for assigning modes to periodicities. However, the relatively simple tests of this paper have been sufficient to confirm that regular period spacings in  $g$ -mode sdB pulsators exist and provide useful constraints which stellar models can now aspire to fit.

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## 6 ON-LINE MATERIAL



**Table 4.** Periods and period spacings for KIC2697388. Identifications (column 1) and periods are those from Paper III (Reed et al. 2010). Columns 3, 4, and 5 provide the mode degree  $\ell$  and the overtone fit to  $P_\ell = P_{\ell_0} + n \cdot \Delta P_\ell$  where  $n$  is arbitrarily chosen such that there are no negative values. Column 6 provides the difference between the observed and asymptotic relation period, column 7 lists the fractional period differences and column 8 is the observed spacing  $P(n_{\ell,i}) - P(n_{\ell,j})/(i - j)$ . It is ambiguous whether  $\ell = 1$  or 2 modes should be associated with f25, f23, and f11. f25 was not used for the  $\ell = 1$  fit as it is most likely  $\ell = 2$ .

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f36	1124.964	1	0		-0.373	-0.002	-
f35	1279.784	-	-	-	-	-	-
f34	1349.752	1	1		-15.680	-0.065	224.8
f34	1349.752	2		5	-2.578	0.019	-
f33	1958.219	-	-	-	-	-	-
f32	1960.990	-	-	-	-	-	-
f31	1963.985	-	-	-	-	-	-
f30	2757.118	2	-	15	19.393	0.140	140.7
f29	3008.732	2	-	17	-6.072	0.044	125.8
f28	3517.219	1	10	-	-8.790	-0.037	254.2
f27	3700.224	2	-	22	-7.277	-0.053	138.3
f26	3757.261	1	11	-	-8.815	-0.037	240.0
f25	3980.103	1	12	-	-26.040	-0.108	273.0
f25	3980.103	2	-	24	-4.477	0.032	139.9
f24	4125.623	2	-	25	2.503	0.018	145.5
f23	4253.082	1	13	-	6.872	0.029	247.9
f23	4253.082	2	-	26	-8.577	-0.062	127.5
f22	4536.116	2	-	28	-2.622	-0.019	141.5
f21	4750.550	1	15	-	24.206	0.101	248.7
f20	4983.073	1	16	-	16.661	0.069	232.5
f19	5374.965	2	-	34	4.990	0.036	139.8
f18	5506.391	2	-	35	-2.123	-0.015	131.4
f17	5679.474	1	19	-	-7.139	-0.030	232.1
f16	5922.022	1	20	-	-4.658	-0.019	242.5
f15	6393.989	-	-	-	-	-	-
f14	6398.864	1	22	-	-7.951	-0.033	238.4
f13	6627.321	2	-	43	10.491	0.076	140.1
f12	6637.021	1	23	-	-9.861	-0.041	238.2
f11	6895.323	1	24	-	8.404	0.035	258.3
f11	6895.323	2	-	45	1.414	0.010	134.0
f10	7043.651	2	-	46	11.202	0.081	148.3
f9	7294.763	2	-	48	-14.764	-0.107	125.6
f8	7607.932	1	27	-	0.781	0.003	237.5
f7	7791.189	-	-	-	-	-	-
f6	8312.182	1	30	-	-15.170	-0.063	234.8
f5	8367.893	-	-	-	-	-	-
f4	8799.008	1	32	-	-8.479	-0.035	243.4
f3	8970.500	2	-	60	-1.501	-0.011	139.6
f2	10248.154	1	38	-	0.264	0.001	241.5
f1	11222.207	1	42	-	14.048	0.059	243.5

**Table 5.** Same as Table 4 for KIC2991403. Identifications (column 1) and periods are those from Paper V (Kawaler et al. 2010b).

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f16	2709.9	1	0	-	1.328	0.005	-
f15	2981.4	1	1	-	4.304	0.016	271.5
f14	2986.6	2	-	9	10.959	0.071	-
f13	2991.8	-	-	-	-	-	-
f12	3233.3	-	-	-	-	-	-
f11	3244.8	1	2	-	0.820	0.003	251.9
f10	3374.7	-	-	-	-	-	-
f9	3504.9	1	3	-	-9.244	-0.034	260.1
f8	3519.1	-	-	-	-	-	-
f7	3693.5	-	-	-	-	-	-
f6	3781.5	1	4	-	-1.167	-0.004	276.7
f5	4326.6	1	6	-	6.885	0.026	272.6
f4	4337.3	2	-	18	-22.908	-0.149	150.1
f3	5124.0	1	9	-	-1.286	-0.005	265.8
f2	5136.5	2	-	23	7.088	0.046	159.8
f1	6365.0	2	-	31	4.861	0.032	153.6

**Table 6.** Same as Table 4 for KIC3527751. Identifications (column 1) and periods are those from Paper III (Reed et al. 2010). Periods f25, f23, f5, f4, f3, and f2 are listed twice as they could be associated with either  $\ell = 1$  or 2 modes. We suggest that f23 is  $\ell = 1$  and that f5, f4, f3, and f2 are all  $\ell = 2$  modes and they were not used in the  $\ell = 1$  fit.

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f38	984.427	1	0	-	7.425	0.028	-
f37	997.536	-	-	-	-	-	-
f36	1072.455	-	-	-	-	-	-
f35	1118.081	-	-	-	-	-	-
f34	1290.822	-	-	-	-	-	-
f33	1334.717	2	-	5	-8.401	-0.055	-
f32	1344.234	-	-	-	-	-	-
f31	1344.720	-	-	-	-	-	-
f30	1389.787	-	-	-	-	-	-
f29	1506.672	1	2	-	-2.538	-0.010	261.123
f28	1765.523	1	3	-	-9.790	-0.037	258.851
f27	1838.307	-	-	-	-	-	-
f26	2264.793	2	-	11	0.243	0.002	155.013
f25	2571.681	1	5	-	-1.943	-0.007	268.719
f25	2571.681	2	-	13	-0.012	0.001	153.444
f24	2728.982	2	-	14	3.717	0.024	157.301
f23	3633.516	1	10	-	-4.523	-0.017	265.459
f23	3633.516	2	-	20	-13.181	-0.086	150.756
f22	3799.665	2	-	21	-0.604	0.004	166.149
f21	3811.326	-	-	-	-	-	-
f20	3910.932	1	11	-	6.789	0.026	277.416
f19	3951.085	2	-	22	-2.756	-0.018	151.420
f18	4263.556	2	-	24	2.571	0.017	156.236
f17	4574.895	2	-	26	6.767	0.044	155.670
f16	4588.472	-	-	-	-	-	-
f15	4702.076	1	14	-	-0.378	-0.001	263.715
f14	4719.823	2	-	27	-1.877	-0.012	144.928
f13	4875.586	2	-	28	0.314	0.002	155.763
f12	5523.277	1	17	-	22.512	0.085	273.734
f11	5816.402	2	-	34	19.698	0.128	156.803
f10	5956.947	2	-	35	6.671	0.043	140.545
f9	6254.758	2	-	37	-2.662	-0.017	148.906
f8	6285.436	1	20	-	-13.640	-0.051	254.053
f7	7091.397	1	23	-	-5.990	-0.023	268.654
f6	7365.565	1	24	-	2.075	0.008	274.168
f5	7932.131	1	26	-	36.433	0.137	283.283
f5	7932.131	2	-	48	-14.580	-0.095	152.488
f4	8415.768	1	28	-	-12.137	-0.046	241.819
f4	8415.768	2	-	51	8.341	0.054	161.212
f3	8724.069	1	29	-	30.060	0.113	308.301
f3	8724.069	2	-	53	9.499	0.062	154.151
f2	9480.648	1	32	-	-11.672	-0.044	252.193
f2	9480.648	2	-	58	-1.782	-0.012	151.316
f1	10852.611	2	-	67	-11.966	-0.078	152.440

**Table 7.** Same as Table 4 for KIC5807616 (KPD1943). Identifications (column 1) and periods are those from Paper III (Reed et al. 2010). We include the *suggested* frequencies of Paper III. Periods f14, f10, f9, and f6 are listed twice as they could be associated with either  $\ell = 1$  or 2 modes. We suggest that f14 and f9 are  $\ell = 2$ . The last two columns provide the associated  $\ell$  and  $n$  values from van Grootel et al. (2010).

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)	VVG ID $\ell$	$n$
s25	2346.420	-	-	-	-	-	-	2	15
f18	2475.513	2	-	16	2.381	0.017	-	2	16
f17	2540.590	-	-	-	-	-	-	1	9
s24	2765.289	-	-	-	-	-	-	4	34
f16	2774.381	-	-	-	-	-	-	2	18
f15	3162.719	2	-	21	-6.063	-0.044	137.44	1	11
f14	3314.876	1	12	-	16.512	0.068	-	2	22
f14	3314.876	2	-	22	6.964	0.050	152.16	2	22
s23	3446.011	2	-	23	-1.031	-0.008	131.14	2	23
f13	3786.345	1	14	-	43.744	0.015	235.73	1	14
f12	4002.153	2	-	27	-1.050	-0.008	139.13	1	15
f11	4027.352	1	15	-	2.632	0.011	241.01	2	27
f10	4272.329	1	16	-	5.490	0.023	244.98	2	29
f10	4272.329	2	-	29	-9.494	-0.068	134.91	2	29
s22	4426.714	2	-	30	5.761	0.041	141.40	2	30
f9	4963.086	1	19	-	-30.109	-0.124	230.25	2	34
f9	4963.086	2	-	34	-14.387	-0.103	134.09	2	34
f8	5012.561	-	-	-	-	-	-	1	19
f7	5726.085	1	22	-	6.534	0.027	254.33	1	22
f6	5958.808	1	23	-	-2.862	-0.012	232.72	2	41
f6	5958.808	2	-	41	7.424	0.053	142.25	2	41
f5	6198.841	1	24	-	-4.948	-0.020	240.03	1	24
f4	6678.527	1	26	-	-9.499	-0.039	239.84	2	46
f3	7055.779	-	-	-	-	-	-	2	49
f2	8372.266	1	33	-	-10.591	-0.044	241.96	2	58
f1	9132.311	1	36	-	23.098	0.095	253.35	1	36

**Table 8.** Same as Table 4 for KIC7664467 Identifications (column 1) and periods are those from Paper VII (Baran et al. 2010a).

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f1	4050.680	2	-	11	14.96	0.10	-
f2	4135.197	1	0	-	11.212	0.043	-
s7	4640.562	1	2	-	-3.464	-0.013	252.68
f3	5156.458	1	4	-	-7.609	-0.029	257.95
f4	5688.625	1	6	-	4.517	0.017	266.08
f5	7487.792	1	13	-	-16.459	-0.063	257.02
f6	9076.175	1	19	-	11.801	0.045	264.73

**Table 9.** Same as Table 4 for KIC7668647 Identifications (column 1) and periods are those from Paper VII (Baran et al. 2010a). s21 is not listed in Paper VII, but is suggested by us. Periods f13 and f5 are listed twice as they could either  $\ell = 1$  or 2 modes.

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f18	2891.88	1	0	-	-11.11	-0.045	-
f17	3387.730	2	-	12	1.76	0.012	-
s21	3392.21	1	2	-	-7.07	-0.028	250.17
f16	3820.65	2	-	15	0.56	0.004	144.31
f15	4112.56	2	-	17	3.06	0.021	145.96
f14	4146.91	1	5	-	3.19	0.013	251.57
f13	4414.39	1	6	-	22.52	0.091	267.48
f13	4414.39	2	-	19	15.474	0.107	150.92
f12	4889.65	1	8	-	1.48	0.006	237.63
f11	4967.24	-	-	-	-	-	-
f10	5099.84	2	-	24	-22.61	-0.156	132.60
f9	5140.81	1	9	-	4.49	0.018	251.16
f8	5288.87	-	-	-	-	-	-
f7	5877.88	1	12	-	-2.89	-0.012	245.69
f6	6288.29	2	-	32	8.179	0.057	148.56
f5	6863.01	1	16	-	-10.34	-0.042	246.29
f5	6863.01	2	-	36	4.08	0.028	143.68
f4	7605.87	1	19	-	-11.92	-0.048	247.62
f3	8367.25	1	22	-	5.01	0.020	253.79
f2	8629.77	1	23	-	19.386	0.078	262.52
f1	9093.93	1	25	-	-12.75	-0.051	232.08

**Table 10.** Same as Table 4 for KIC8302197. Identifications (column 1) and periods are those from Paper VII (Baran et al. 2010a). Suggested frequencies are not listed in Paper VII.

ID	Period (sec)	$\ell$	$n_1$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f7	3269.75	1	0	-8.17	-0.032	-
f6	3530.54	1	1	-5.09	-0.020	260.78
s9	4577.5	1	5	11.07	0.043	261.74
f5	5347.92	1	8	8.39	0.033	256.81
f4	5607.43	1	9	10.19	0.040	259.50
f3	6109.39	1	11	-3.25	-0.013	250.98
f2	6868.71	1	14	-17.02	-0.066	253.11
s8	7403.6	1	16	2.47	0.010	267.45
f1	7917.94	1	18	1.41	0.005	257.17

**Table 11.** Same as Table 4 for KIC9472174 (2M1938+4603). Identifications (column 1) and periods are those from Østensen et al. (2010c). fA20 is also listed as  $\ell = 2$  though  $\ell = 1$  is a better fit.

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
fA20	2158.441	1	0	-	-8.469	-0.033	-
fA20	2158.441	2	-	6	18.184	0.123	-
fA19	2303.443	2	-	7	15.490	0.105	145.00
fA18	2692.512	1	2	-	14.348	0.056	267.04
fA17	2824.900	-	-	-	-	-	-
fA16	2951.098	1	3	-	17.307	0.068	258.59
fA15	3134.262	2	-	13	-39.867	-0.270	138.47
fA14	3418.37	-	-	-	-	-	-
fA13	3456.036	1	5	-	10.991	0.043	252.47
fA12	4444.656	1	9	-	-22.897	-0.090	247.16
fA11	4567.474	-	-	-	-	-	-
fA10	4634.736	2	-	23	-16.353	-0.111	150.05
fA9	5474.256	1	13	-	-15.805	-0.090	257.40
fA8	5853.966	2	-	31	21.309	0.144	152.40
fA7	6577.086	2	-	36	5.948	0.040	144.62
fA6	7452.603	2	-	42	-4.711	0.032	145.92
fA5	8557.319	1	25	-	-0.266	0.001	256.92
fA4	10208.828	2	-	61	-54.710	-0.370	145.06
fA3	10915.351	-	-	-	-	-	-
fA2	17253.692	1	59	-	4.790	0.019	255.78
fA1	19884.934	2	-	121	21.155	0.143	148.86

**Table 12.** Same as Table 4 for KIC10001893. Identifications (column 1) and periods are those from Paper VII (Baran et al. 2010a). Periods shortward of f14 can be associated with different modes using slightly different period spacings. Both results are provided with Scheme 2 having a slightly better fit. Suggested periods s27, s28, and s29 were detected using a rough fit to a wavelet analysis and require further work to determine their validity.

ID	Period (sec)	Scheme 1						Scheme 2					
		$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f26	1023.05	1	0	-	13.35	0.050	-	2	-	3	0.24	0.001	-
f25	1472.43	2	-	6	-26.14	-0.169	-	1	2	-	-19.57	-0.127	-
f24	2555.15	2	-	13	-24.65	-0.160	154.67	1	6	-	-10.99	-0.071	270.68
f23	2780.21	2	-	14	45.96	0.298	225.07	-	-	-	-	-	-
f22	3086.54	2	-	16	43.35	0.281	153.16	1	8	-	-16.66	-0.108	265.70
f21	3348.33	2	-	18	-3.77	-0.024	130.90	2	-	18	4.36	0.028	155.02
f20	3496.20	2	-	19	-10.36	-0.067	147.87	2	-	19	-2.51	-0.016	147.87
f19	3645.63	1	10	-	-24.12	0.091	262.26	1	10	-	5.373	0.020	279.55
f18	3802.95	2	-	21	-12.54	-0.081	153.38	2	-	21	-5.25	-0.016	153.38
f17	3919.22	1	11	-	-16.53	0.062	273.59	1	11	-	10.43	0.039	273.59
f16	4746.35	1	14	-	12.59	0.047	275.71	2	-	27	9.87	0.036	130.96
f15	4884.89	2	-	28	-11.84	-0.077	154.56	2	-	28	-6.53	-0.042	154.56
f14	5022.31	1	15	-	22.54	0.085	275.96	1	15	-	39.39	0.148	275.77
s29	5038	2	-	29	-13.18	-0.085	153.12	2	-	29	-8.15	-0.053	153.12
f13	5288.98	1	16	-	23.20	0.087	266.66	1	16	-	37.52	0.141	266.66
f12	5540.21	1	17	-	8.43	0.032	251.23	1	17	-	20.21	0.076	251.23
f11	5792.81	1	18	-	-4.98	-0.019	252.60	1	18	-	4.28	0.016	252.60
s28	5803	2	-	34	-20.49	-0.133	154.00	2	-	34	-16.9	-0.109	153.0
s27	5956	2	-	35	-21.96	-0.142	153.00	2	-	35	-18.62	-0.121	153.0
f10	6049.56	1	19	-	-14.22	-0.053	256.76	1	19	-	-7.49	-0.028	256.76
f9	6329.06	1	20	-	-0.73	-0.003	279.50	1	20	-	3.48	0.013	279.50
f8	6576.60	1	21	-	-19.19	-0.072	247.54	1	21	-	-17.52	-0.66	247.54
f7	6845.23	1	22	-	-16.57	-0.062	268.63	1	22	-	-17.42	-0.065	268.63
f6	7112.78	1	23	-	-15.02	-0.056	267.14	1	23	-	-18.40	-0.069	267.55
f5	7388.93	1	24	-	-4.897	-0.018	276.14	1	24	-	-10.80	-0.041	276.14
f4	7659.85	1	25	-	0.03	0.001	270.92	1	25	-	-8.41	-0.032	270.92
f3	7942.18	1	26	-	16.37	0.062	282.34	1	26	-	5.40	0.020	282.34
f2	8743.58	1	29	-	19.74	0.074	267.13	1	29	-	1.19	0.004	267.13
f1	12899.51	-	-	-	-	-	-	-	-	-	-	-	-

**Table 13.** Same as Table 4 for KIC10553698. Identifications (column 1) and periods are those from Paper VII (Baran et al. 2010a). Periods f36, f30, f20, f15, and f8 could be matched by either  $\ell = 1$  or 2, though f30 and f9 are most likely  $\ell = 1$  and f9 was not used in the  $\ell = 2$  fit.

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f36	2238.30	1	0	-	8.17	0.030	-
f36	2238.30	2	-	6	6.78	0.043	-
f35	2382.81	2	-	7	1.74	0.011	144.50
f34	2518.16	1	1	-	16.88	0.062	279.86
f33	2545.22	2	-	8	-5.40	-0.034	162.41
f32	2759.15	1	2	-	-13.28	-0.049	240.99
f31	2867.13	2	-	10	12.35	0.079	160.96
f30	3036.87	1	3	-	-6.715	-0.025	277.72
f30	3036.87	2	-	11	21.95	0.140	169.74
f29	3124.89	-	-	-	-	-	-
f28	3294.19	2	-	13	-30.17	-0.193	142.35
f27	3308.79	1	4	-	-5.966	-0.025	271.91
f26	3447.91	-	-	-	-	-	-
f25	3482.53	2	-	14	-2.43	-0.015	188.35
f24	3587.68	1	5	-	1.78	0.007	278.89
f23	3611.47	2	-	15	-33.17	-0.193	128.916
f22	3859.72	1	6	-	2.68	0.010	272.05
f21	4058.55	-	-	-	-	-	-
f20	4123.14	1	7	-	-5.05	0.019	263.42
f20	4123.14	2	-	18	11.47	0.073	170.56
f19	4392.03	1	8	-	-7.31	-0.027	268.89
f18	4673.52	1	9	-	3.03	0.011	281.49
f17	4806.31	-	-	-	-	-	-
f16	4951.42	1	10	-	9.77	0.036	277.89
f15	5202.87	1	11	-	-9.93	-0.037	251.45
f15	5202.87	2	-	25	-5.57	-0.036	159.14
f14	5384.23	2	-	26	19.12	0.122	161.162
f13	5703.13	2	-	28	24.66	0.157	159.45
f12	5955.79	-	-	-	-	-	-
f11	6223.85	-	-	-	-	-	-
f10	6473.08	2	-	33	11.21	0.072	153.99
f9	6742.50	2	-	35	-32.72	-0.209	154.08
f8	7116.77	1	18	-	5.93	0.022	273.42
f8	7116.77	2	-	37	28.19	0.180	160.92
f7	7269.10	2	-	38	23.84	0.152	156.60
f6	7765.49	-	-	-	-	-	-
f5	8330.75	2	-	45	-11.28	-0.072	151.66
f4	8591.40	-	-	-	-	-	-
f3	9115.72	2	-	50	-9.70	-0.062	157.00
f2	9384.20	-	-	-	-	-	-
f1	9588.52	2	-	53	-6.95	-0.044	157.60



**Table 14.** Same as Table 4 for KIC010670103. Identifications (column 1) and periods are those from Paper III (Reed et al. 2010).

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f28	4920.1	2	-	8	3.6	0.024	-
f27	5061.6	2	-	9	-0.5	-0.004	141.5
f26	5208.5	2	-	10	0.7	0.005	146.9
f25	5353.1	2	-	11	-0.2	-0.002	144.6
f24	5493.3	2	-	12	-5.6	-0.039	140.2
f23	6081.9	2	-	16	0.6	0.004	147.2
f22	6484.6	1	0	-	-17.8	-0.071	-
f21	6996.0	1	2	-	-8.6	-0.034	255.7
f20	7106.8	2	-	23	6.3	0.044	146.4
f19	7241.2	2	-	24	-4.9	-0.033	134.4
f18	7515.8	1	4	-	8.9	0.035	259.9
f17	7761.7	1	5	-	3.7	0.015	245.9
f16	8259.5	1	7	-	-0.8	-0.003	248.9
f15	8758.8	1	9	-	-3.8	-0.015	249.7
f14	9012.6	1	10	-	-1.1	-0.004	253.8
f13	9265.8	1	11	-	1.0	0.004	253.2
f12	9506.1	1	12	-	-9.8	-0.039	240.3
f11	10271.0	1	15	-	1.7	0.007	255.0
f10	10532.7	1	16	-	12.2	0.049	261.7
f9	11316.3	1	19	-	42.44	0.169	261.2
f8	11533.7	1	20	-	8.7	0.035	217.4
f7	12278.2	1	23	-	-0.2	-0.001	248.2
f6	12788.2	1	25	-	7.6	0.030	255.0
f5	13019.7	1	26	-	-12.1	-0.048	231.5
f4	13262.8	1	27	-	-20.1	-0.080	243.1
f3	13779.6	1	29	-	-5.6	-0.022	258.4
f2	15343.7	-	-	-	-	-	-
f1	16290.1	1	39	-	-6.4	-0.025	251.1

**Table 15.** Same as Table 4 for KIC11179657. Identifications (column 1) and periods are those from Paper V (Kawaler et al. 2010b). f1 is also listed as  $\ell = 2$  for completeness although it is most likely  $\ell = 1$ .

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f11	2844.27	2	-	6	4.38	0.033	-
f10	2956.26	-	-	-	-	-	-
f9	2965.95	2	-	7	-7.58	-0.057	121.68
f8	3240.38	2	-	9	-0.44	-0.003	137.215
f7	3381.00	2	-	10	6.54	0.049	140.62
f6	3503.48	2	-	11	-5.03	-0.038	122.48
f5	3513.35	1	0	-	0.01	0.001	-
f4	3522.81	-	-	-	-	-	-
s12	4314.70	2	-	17	4.72	0.035	135.2
f3	5109.26	2	-	23	-2.59	-0.019	132.4
f2	5130.40	1	7	-	-0.09	-0.001	231.0
f1	5361.58	2	-	25	-18.35	-0.137	126.16
f1	5361.58	1	8	-	0.07	0.001	231.18

**Table 16.** Same as Table 4 for KIC11558725. Identifications (column 1) and periods are those from Paper VII (Baran et al. 2010a).

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f44	1403.78	2	-	0	2.05	0.014	-
f43	1406.85	-	-	-	-	-	-
f42	1519.33	2	-	1	-24.97	-0.175	-115.55
f41	2558.20	2	-	8	15.90	0.112	148.41
f40	2613.64	-	-	-	-	-	-
f39	2741.74	-	-	-	-	-	-
s55	2834.64	2	-	10	7.20	0.051	138.22
f38	2855.62	-	-	-	-	-	-
f37	3100.84	-	-	-	-	-	-
f36	3116.80	2	-	12	4.22	0.030	139.65
f35	3211.96	-	-	-	-	-	-
f34	3250.93	2	-	13	-4.22	-0.030	134.13
f33	3276.54	-	-	-	-	-	-
f32	3377.28	-	-	-	-	-	-
f31	3388.92	-	-	-	-	-	-
f30	3530.80	2	-	15	-9.49	-0.067	139.94
f29	3640.52	1	5	-	-20.81	-0.084	-
f28	3686.12	2	-	16	3.26	0.023	155.32
f27	3822.21	2	-	17	-3.12	-0.022	136.20
f26	3969.80	2	-	18	1.80	0.013	147.48
f25	4160.37	1	7	-	5.51	0.022	260.66
f24	4359.19	-	-	-	-	-	-
f23	4405.37	2	-	21	9.65	0.068	145.19
f22	4421.03	1	8	-	19.40	0.079	260.66
f21	4663.89	1	9	-	15.49	0.063	242.86
f21	4663.89	2	-	23	-16.97	-0.119	129.26
f20	4908.04	1	10	-	12.87	0.052	244.15
f19	5150.71	1	11	-	8.78	0.036	242.68
f18	5375.06	1	12	-	-13.64	-0.055	224.352
f17	5610.76	1	13	-	-24.72	-0.100	236.82
f16	5675.24	-	-	-	-	-	-
f15	5869.32	1	14	-	-12.92	-0.052	258.56
f14	6106.13	-	-	-	-	-	-
f13	6371.19	1	16	-	-4.60	-0.019	265.05
f13	6371.19	2	-	35	-20.529	-0.144	142.28
f12	6552.55	2	-	36	18.27	0.128	143.15
f11	6626.11	1	17	-	3.57	0.014	254.93
f10	6892.92	1	18	-	23.60	0.096	266.80
f9	7121.59	1	19	-	5.52	0.022	228.68
f9	7121.59	2	-	40	17.03	0.119	150.08
s54	7354.49	1	20	-	-8.354	-0.034	232.90
f8	7624.98	1	21	-	15.36	0.062	270.48
f7	8101.45	1	23	-	-1.70	-0.007	238.24
f7	8101.45	2	-	47	-1.11	-0.008	140.81
f6	8612.79	1	25	-	16.10	0.065	255.67
f5	9567.17	1	29	-	-16.60	-0.067	238.60
f4	9900.75	-	-	-	-	-	-
f3	9990.93	-	-	-	-	-	-
f2	11094.93	1	35	-	42.168	0.171	254.36
f2	11094.93	2	-	68	-1.61	-0.011	142.55
f1	12798.82	1	42	-	21.04	0.085	243.197
f1	12798.82	2	-	80	-8.58	-0.060	141.99

**Table 17.** Same as Table 4 for KPD0629-0016. Identifications (column 1) and periods are those from Charpinet et al. (2010). Periods f8 and f12 are shown for both  $\ell = 1$  and 2.

ID	Period (sec)	$\ell$	$n_1$	$n_2$	$\delta P$ (sec)	$\delta P/\Delta P$	Spacing (sec)
f7	2601.915	1	0	-	-16.86	-0.068	-
f3	2838.557	1	1	-	-27.38	-0.111	236.64
f5	3355.602	1	3	-	-4.68	-0.019	258.52
f9	3513.892	2	-	14	1.89	0.013	-
f2	3614.638	1	4	-	7.18	0.029	259.04
f6	3662.012	2	-	15	7.26	0.051	148.12
f8	4360.573	1	7	-	11.60	0.047	248.645
f8	4360.573	2	-	20	-7.88	-0.055	139.71
f4	4618.324	1	8	-	22.18	0.090	257.751
f1	4871.190	1	9	-	27.89	0.113	252.866
f10	4878.150	-	-	-	-	-	-
f15	6820.557	1	17	-	-0.13	-0.001	243.67
f13	7301.076	1	19	-	-13.95	-0.056	240.26
f11	7640.621	2	-	43	-10.87	-0.076	142.61
f14	8304.588	1	23	-	0.88	0.004	250.88
f16	8477.007	-	-	-	-	-	-
f12	8803.020	1	25	-	4.97	0.020	249.22
f12	8803.020	2	-	51	9.60	0.067	145.30
f17	10516.563	1	32	-	-11.67	-0.047	244.79