Modelling *Kepler* observations of solar-like oscillations in the red-giant star HD 186355

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ABSTRACT

We have analysed oscillations of the red giant star HD 186355 observed by the NASA Kepler satellite. The data consist of the first five quarters of science operations of Kepler, which covers around 13 months. The high-precision time-series data allow us to accurately extract the oscillation frequencies from the power spectrum. We find the frequency of the maximum oscillation power, $\nu_{\text{max}}$, and the mean large frequency separation, $\Delta \nu$, are around 106 and 9.4 $\mu$Hz respectively. A regular pattern of radial and non-radial oscillation modes is identified by stacking the power spectra in an échelle diagram. We use the scaling relations of $\Delta \nu$ and $\nu_{\text{max}}$ to estimate the preliminary asteroseismic mass, which is confirmed with the modelling result ($M = 1.43 \pm 0.02 M_\odot$) using the Yale Rotating stellar Evolution Code (YREC7). In addition, we constrain the effective temperature, luminosity and radius from comparisons between observational constraints and models. A number of mixed $l = 1$ modes are also detected and taken into account in our model comparisons. We find a mean observational period spacing for these mixed modes of about 58 sec, suggesting that this red giant branch star is in the shell hydrogen-burning phase.

Subject headings: stars: individual (HD 186355) - stars: oscillations - stars: modelling
1. Introduction

Studying solar-like oscillations provides us a powerful method to probe the interiors of stars (Christensen-Dalsgaard 2004). Solar-like oscillations are expected in low-mass main-sequence stars cooler than the red edge of the classical instability strip in the HR diagram (Christensen-Dalsgaard 1982; Christensen-Dalsgaard & Frandsen 1983; Houdek et al. 1999), as well as in more evolved red giants which represent the future of our own Sun (Dziembowski et al. 2001; Dupret et al. 2009). This is because the turbulent convective motions near the surface are stochastically exciting oscillations.

Asteroseismology of red giants has developed rapidly. It began with several detections of solar-like oscillations in G and K-type giants based on ground-based observations in both radial velocity (Frandsen et al. 2002; De Ridder et al. 2006) and photometry (Stello et al. 2007) and on space-based photometry detections observed by the Microvariability and Oscillations of Stars (MOST; Barban et al. 2007; Kallinger et al. 2008a,b), Wide Field Infrared Explorer (WIRE; Buzasi et al. 2000; Retter et al. 2003; Stello et al. 2008), Hubble Space Telescope (HST; Edmonds & Gilliland 1996; Gilliland 2008; Stello & Gilliland 2009), and Solar Mass Ejection Imager (SMEI; Tarrant et al. 2007). The oscillation periods in red giants range from hours to days. Ground-based observations usually suffer from interruptions and aliasing which complicate the measurement of oscillations. On the other hand, observations from space can provide high signal-to-noise ratio (SNR) and continuous data sets from which we may extract the oscillation parameters accurately. The 150-day long observations by the Convection Rotation and planetary Transits satellite (CoRoT) clearly detected radial as well as non-radial oscillations in the range from 10-100 μHz (De Ridder et al. 2009; Hekker et al. 2009; Carrier et al. 2010) and greatly increased the number of detected pulsating G and K giants, which led to a huge breakthrough in the study of red giants. These observations were followed by even more impressive results by Kepler.
This paper presents observations and models of HD 186355 (HIP 96878, KIC 11618103), which is one of the brightest red giants in the Kepler field (V = 7.95).

2. Observations

The Kepler Mission (Borucki et al. 2008, 2010) was successfully launched on March 7, 2009. The primary scientific goal of Kepler is to search for Earth-size planets in or near the habitable zone and to determine how many stars have this kind of planets in our Milky Way. Kepler is equipped with a 0.95-meter diameter telescope with an array of CCDs which continuously points to a large area of the sky in the constellations Cygnus and Lyra to detect the transits of the planets. Over the whole course (at least 3.5 years) of the mission, the spacecraft will simultaneously measure the variations in the brightness of more than 100,000 stars, which will be outstanding data for the study of asteroseismology. For many of these stars we can detect solar-like oscillations which will allow us to investigate the stars in detail and obtain their fundamental properties, by using the techniques of asteroseismology (Christensen-Dalsgaard et al. 2007; Aerts et al. 2010).

We used the first five quarters of data of HD 186355, which covers a total of around 13 months. The raw long-cadence data (29.4 minutes sampling; Jenkins et al. 2010), which we used in this paper, were corrected by performing a point-to-point sigma clipping to remove the outliers. Additionally, a thermal drift was corrected by fitting a second-order polynomial to the affected parts of the time-series. From the parallax of 5.44 ± 0.63 mas (van Leeuwen 2007) and using a bolometric correction for G5 giants of -0.34 from Kaler (1989), we derived the luminosity of the star to be 24.0 ± 5.6 $L_\odot$. We took the effective
temperature \( T_{\text{eff}} = 4867 \pm 150 \text{ K} \) from *Kepler* Input Catalogue (KIC; Brown et al. 2011).

### 3. Global oscillation analysis

Solar-like oscillations are high-order and low-degree p-modes, and the observed frequencies (usually acoustic oscillation modes, namely p-modes) are regularly spaced, described by the asymptotic relation (Tassoul 1980; Gough 1986):

\[
\nu_{nl} \approx \Delta \nu (n + \frac{1}{2} l + \epsilon) - l(l + 1) D_0, \tag{1}
\]

where \( n \) is the radial order and \( l \) is the angular degree. \( \Delta \nu \) (large frequency separation) is approximately the inverse of the sound travel time across the star, while \( \epsilon \) is sensitive to the surface layers and, for relatively unevolved stars, \( D_0 \) is sensitive to the sound speed gradient near the core. As the star evolves, the stellar envelope starts to expand and the p-mode frequencies gradually decrease while oscillations in the core driven by buoyancy (g-modes) shift to higher frequencies. This eventually leads to so-called "mixed modes". These are non-radial oscillation modes that have a mixed character, behaving like g-modes in the core and p-modes in the envelope, and shifting in frequency as they undergo the so-called *avoided crossings* (Osaki 1975; Aizenman et al. 1977). For red giants, the \( l = 1 \) modes in particular depart from the relation due to many avoided crossings (Huber et al. 2010; Mosser et al. 2011).

Our frequency analysis covers three basic steps that are performed on the power spectrum of the *Kepler* light curve: fitting and correcting for the background; estimating the frequency of maximum power \( (\nu_{\text{max}}) \) and the large separation \( (\Delta \nu) \); and extracting individual frequencies \( (\nu_{nl}) \). In the following subsections, we will describe the three analysis steps in detail.
3.1. Modelling the Background and $\nu_{\text{max}}$

The power spectrum shows a non-white frequency-dependent background signal due to stellar activity, granulation and faculaes (Karoff 2008). The background can be modelled by a sum of several Lorentzian-like functions (Harvey 1985), which represent those physical processes separately that are believed to be strongly connected to the turbulent activities in the stellar convective envelope. The background signal and the white noise were fitted to the smoothed power spectrum. The smoothed power spectrum was obtained by using a Gaussian with full width half maximum (FWHM) of $3 \Delta \nu$, where $\Delta \nu$ was estimated directly from the power spectrum. The power spectrum shows a typical solar-like pattern with a large separation of $\Delta \nu = 9.37 \mu \text{Hz}$. Stellar activity, Granulation and faculaes were represented by modified Lorentzian-like functions first introduced by Karoff (2008), which can reflect the physical properties of the background more realistically than the Harvey model with a constant slope of 2. This background model has a shallower slope at low frequencies and a steeper slope at higher frequencies, corresponding to turbulence and granulation, respectively. The power excess hump from stellar oscillations is approximately Gaussian, so the complete spectrum was modeled by:

$$P(\nu) = P_n + \sum_{i=1}^{3} \frac{4\sigma_i^2 \tau_i}{1 + (2\pi \nu \tau_i)^2 + (2\pi \nu \tau_i)^4} + P_g \exp \left( \frac{-(\nu_{\text{max}} - \nu)^2}{2\sigma_g^2} \right),$$

where $P_n$ corresponds to the white noise component, $\sigma_i$ is the rms intensity of the granules and $\tau_i$ is the characteristic time scale of granulation. For the Gaussian term, the parameters $P_g$, $\nu_{\text{max}}$, and $\sigma_g$ are the height, the central frequency, and the width of the power excess hump.

Fig. 1 shows the power density spectrum of HD 186355, together with the fitted model using Eq. (2) from a least-squares fit. We fitted the three components of the background and the white noise simultaneously to the lightly smoothed spectrum (Gaussian with FWHM of 0.5 $\mu \text{Hz}$) outside the region where the power excess hump is seen. The $\nu_{\text{max}}$ we
obtained from the Gaussian term is $106.5 \pm 0.3 \, \mu Hz$. Finally, the background and the white noise were used to correct the power density spectrum for background signal, leaving only the oscillation signal (lower panel of Fig. 1).

### 3.2. Individual Frequencies

To determine the oscillation frequencies of the star, we used Period04 (Lenz & Breger 2004) based on the Fourier Theorem to extract multiple frequencies from the data. In the background corrected power density spectrum in Fig. 1, we see some peaks appear to be closely spaced. The main feature of these multiple peaks are due to mixed modes (see also Sect. 4, Beck et al. 2011, Bedding et al. 2011) but could also partly be caused by the stellar rotation. The effect of rotation is not considered in this paper. In addition, damping can also split each mode into a series of peaks under a Lorentzian envelope. Therefore, the extracted frequencies from Period04 may be offset from the ‘true’ mode due to oscillation damping, mixed modes or rotation. We did not apply a special selection criteria to choose the ‘right’ mode peaks but simply picked those peaks with high SNR and compared them with the lightly smoothed (FWHM of 0.1 $\mu Hz$) power density spectrum shown in échelle format in Fig 2. To make this diagram, we divided the range of 80 to 140 $\mu Hz$ of the power density spectrum into five segments each $\Delta \nu$ wide. The peaks form the regular ridges that allow us to assign $l$ values. Although the degree of each mode could be recognized directly from Fig. 2, we confirmed the identification by comparing frequencies with the model results in the échelle diagram, as described in Sect. 4. Thanks to the high SNR observed in the mode peaks, we have extracted 33 peaks (SNR bigger than 3) from the power density spectrum (see Table 1), together with their amplitudes and phases. Since the uncertainties derived by Period04 only estimate the internal consistency of the parameters, they are underestimated. We derived more realistic uncertainties (the second column in Table 1)
by means of Monte-Carlo simulations with Period04. The residual time-series \((t, y)\) were obtained by subtracting the sum of multiple sine functions of frequencies \((f_i)_{i=1,2,...,n}\) and the corresponding amplitudes \(A_i\) and phases \(\phi_i\) from the observed time-series \((t, x)\) as

\[
y = x - \sum_{i=1,...,n} A_i \sin(2\pi f_i t + \phi_i).
\]

(3)

Then \(|y|\) is regarded as the observation error of \(x\). We constructed 100 simulated time-series \(z\), which have the same residuals as the observed time-series. The 100 simulated time-series were fitted with the sum of multiple sine functions by taking \((f_i, A_i, \phi_i)_{i=1,2,...,n}\) as initial values according to least-squares algorithm. Hence 100 sets of new \((f_i, A_i, \phi_i)_{i=1,2,...,n}\) were obtained. The standard deviations of each parameter of \((f_i, A_i, \phi_i)_{i=1,2,...,n}\) were then calculated, which were defined as the uncertainty estimates of the parameters.

4. Modelling

The common way to estimate the fundamental properties is to compare calculated model parameters with the observational constraints. We employed the Yale rotating stellar evolution code (YREC; Demarque et al. 2008) for stellar evolution modelling computations, and the non-radial and non-adiabatic stellar pulsation programme JIG developed by Guenther (1994) for frequency calculations. YREC can evolve our models up to the tip of the red giant branch, which is adequate for HD 186355. The input physics of the current YREC version (YREC7) included the latest OPAL opacity tables (Iglesias & Rogers 1996), OPAL equation of state (Rogers & Nayfonov 2002) and NACRE reaction rates (Angulo et al. 1999). At low temperatures, opacities are obtained from Ferguson et al. (2005). Convection is treated under the assumption of mixing length theory (Böhm-Vitense 1958). We did not take rotation, diffusion or convective overshoot into consideration in our calculation.
There are several main inputs in YREC7—mass, $\alpha_{ml}$ (to determine the mixing-length $l_{ml} = \alpha_{ml} H_p$, where $H_p$ is pressure scale height), hydrogen abundance ($X$) and heavy-element abundance ($Z$). Then the best models are searched among those grids after being compared with observation constraints. For our models, $\alpha_{ml}$ and $X$ were fixed to the solar values of 1.8 and 0.72, respectively. The value of $Z$ was varied within a certain range, usually from 0.005 to 0.025 with a step of 0.002, but it changes for models with different masses.

Our initial estimate for the mass was made using scaling relations. Kjeldsen & Bedding (1995) have shown a scaling relation that can be used to predict $\nu_{\text{max}}$ by scaling from the solar case for an arbitrary star:

$$\frac{\nu_{\text{max}}}{\nu_{\text{max},\odot}} \approx \left( \frac{M}{M_{\odot}} \right) \left( \frac{R}{R_{\odot}} \right)^{-2} \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^{-1/2}.$$  \hspace{1cm} (4)

This relation gives a very good estimate for $\nu_{\text{max}}$ for less evolved stars (Bedding & Kjeldsen 2003), while Stello et al. (2008) have shown that it holds also for stars on the giant branch, although with larger uncertainties. Kjeldsen & Bedding (1995) also give the scaling relation to predict $\Delta \nu$

$$\frac{\Delta \nu}{\Delta \nu_{\odot}} \approx \left( \frac{M}{M_{\odot}} \right)^{1/2} \left( \frac{R}{R_{\odot}} \right)^{-3/2}.$$ \hspace{1cm} (5)

Knowing $\nu_{\text{max}}$, $\Delta \nu$ and $T_{\text{eff}}$, the stellar mass is estimated by:

$$\frac{M}{M_{\odot}} \approx \left( \frac{\Delta \nu}{\Delta \nu_{\odot}} \right)^{-4} \left( \frac{\nu_{\text{max}}}{\nu_{\text{max},\odot}} \right)^3 \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^{3/2}.$$ \hspace{1cm} (6)

Finally, since we have obtained $\nu_{\text{max}}$ and $\Delta \nu$ from Sec. 3 and $T_{\text{eff}}$ from KIC, the stellar mass is estimated as $1.41 \pm 0.14 M_{\odot}$. Therefore, the initial masses of our models are chosen to be within the range of 1.25 to 1.55 $M_{\odot}$ with a step of 0.01 $M_{\odot}$.

We looked for models for which the parameters are located inside the 1 $\sigma$ error box confined by the uncertainties of observation results in the H-R diagram. For these sets of modelling parameters, we used a fine resolution for $Z$ (in steps of 0.001) in order to find the
best models. Some models with larger masses were also calculated, with a bigger mass step of 0.1 $M_\odot$, in an attempt to search for models in a large range, because the scaling relations and hence the estimated mass are not so reliable for the giant branch stars.

Fig. 3 shows several evolutionary tracks of models having different input parameters. The rectangle is the 1 $\sigma$ error box whose center locates the observed stellar properties, from which we can see that HD 186355 is on the ascending giant branch, in the shell hydrogen-burning phase. Those models for which the parameters are within the error box and the mean large frequency separations are around 9.37 $\mu$Hz (within 0.03 $\mu$Hz) are indicated by dots. The evolutionary tracks may pass through the same position in the H-R diagram by tuning the inputs. For example, a decrease of mass can be compensated by a decrease of hydrogen and heavy elements abundances to obtain the same position. Taking variations of the mixing-length into consideration, which only move the tracks horizontally but almost have no influence on the luminosity, makes it even more complex to look for models. However, it is beyond the scope of this paper to consider the effects of the mixing-length and hydrogen abundance. We performed a $\chi^2$ minimization to find the best models. We did not find it necessary to apply an offset to the model frequencies to correct for near-surface effects (Kjeldsen et al. 2008). The definition of the function $\chi^2$ was based on two observed results (luminosity and $T_{\text{eff}}$), and individual frequencies. The function $\chi^2$ is defined as follows:

$$
\chi^2 = \left( \frac{T_{\text{eff}} - T'_{\text{eff}}}{150 \text{K}} \right)^2 + \left( \frac{\log L/L_\odot - \log L'/L_\odot}{0.11} \right)^2 + \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\nu_i - \nu'_i}{\sigma_i} \right)^2 ,
$$

where terms with prime are observed results, while $N$ is the number of observed frequencies and $\sigma_i$ is the uncertainty of each frequency listed in Table 1.

We began by fitting only one mode of each degree in each order. For $l = 1$ we took the strongest peak in each order. The results are listed in Table 2. Although tracks of models with masses larger than 1.60 $M_\odot$ also pass through the error box, their oscillation
parameters differ greatly from the observations, which leads to bigger $\chi^2$. From Table 2 we can see the best model has a mass of 1.43 $M_\odot$ and $Z$ of 0.012. However, the difference between this and the 1.60 $M_\odot$ model is very small, which makes both good candidates for the best model. In order to check this result, we plot a series of échelle diagrams in Fig. 4 to compare the theoretical and observed frequencies for each model shown in Table 1. Those frequencies having minima of mode inertias for each degree (see Fig. 5) are picked out as theoretical modes, because those modes with low mode inertias have the highest amplitude at the stellar surface. Compared to models with 1.43 $M_\odot$, models with larger masses do not reproduce the observed frequencies as well and the differences increase with the mass. However, the 1.60 $M_\odot$ model is an exception which produce a rather good match to observed frequencies, for $l = 0$ and 2 modes. The location of the 1.60 $M_\odot$ model is close to the center of error box in HR diagram which, combined with the relatively good fit to the $l = 0$ and 2, leads to a small $\chi^2$ result.

Moreover, we also considered models with masses around the predicted value of 1.43 $M_\odot$, which show échelle diagrams with similar patterns to each other consistent with the $\chi^2$ results. After being compared with the theoretical modes in échelle diagrams, degrees of observed modes are confirmed, including two modes with $l = 3$.

We found multiple oscillation peaks per order for $l = 1$ (see Table 1) from the power spectrum analysed by Period04. As discussed in Sect. 3, these $l = 1$ modes are believed to be mixed modes. Unfortunately, it is usually impossible to observe mixed modes (g-dominated mixed modes) which have very high inertias. However, some of the mixed modes act more like p modes (p-dominated mixed modes), having a lower inertia than the g-dominated mixed modes and hence larger amplitude, which makes them observable. Tassoul (1980) and Miglio et al. (2008) have shown that pure g modes are equally spaced in period. P-dominated mixed modes are also approximately equally spaced in period.
Measuring the period spacing for these observed mixed modes allows us to probe the cores of red giant stars. Beck et al. (2011) have detected mixed modes in a red giant star with Kepler data and measured their period spacing. Subsequently, Bedding et al. (2011) have found a way to distinguish between hydrogen-burning and helium-burning red giants by using their different period spacings. In addition, measuring the period spacing may also provide a method to determine the region of the convective core of those helium-burning red giants. The observed mean period spacing of HD 186355 we obtained is $58 \pm 4$ sec by means of power spectrum of the power spectrum method, which agrees with the value of $56$ sec for this star found by Bedding et al. (2011). According to Bedding et al. (2011), this value indicates that HD 186355 is still in the shell hydrogen-burning phase which agrees with our models.

We took frequencies of those mixed modes which have relatively low theoretical mode inertias for $l = 1$ modes into calculation and obtained $\chi^2_1$ also listed in Table 2. Again the best model is the one with $1.43 \, M_\odot$, but models with masses larger than $1.45 \, M_\odot$ now have a large deviation between observed and theoretical oscillation frequencies. Thus, the potential candidate of our best model the $1.6 \, M_\odot$ model is ruled out after this calculation, because the differences between observed frequencies and theoretical ones for $l = 1$ modes become large when mixed modes are taken into account. Therefore, the model with a mass of $1.43 \, M_\odot$ is adopted as the best model. We searched for models with $\chi^2_1$ smaller than 30 which constrains the resulting mass to be $1.43 \pm 0.02 \, M_\odot$.

5. Conclusion

In this work, we analysed the time series data sets of the star HD 186355 from Kepler to obtain its oscillation parameters. By using the scaling relations between $\Delta \nu$, $\nu_{\text{max}}$ and the stellar effective temperature $T_{\text{eff}}$ we estimated the stellar mass as $1.41 \pm 0.14 \, M_\odot$. In
order to determine the stellar global properties more accurately, we computed a set of models to compare the model results with observational constraints. The best model was found having mass of $1.43 \pm 0.02 \, M_\odot$, which agrees with the scaling value, and $Z$ of 0.012. Furthermore, parameters such as age, effective temperature, luminosity and radius are also determined after comparison (see model with mass of 1.43 in Table 2). We also obtain the mean period spacing of $l = 1$ modes with a value of $58 \pm 4$ sec. From the modelled evolutionary track of HD 186355, we know it is in the shell hydrogen-burning phase, which is consistent with the results of Bedding et al. (2011) on the mean period spacings of mixed modes for red giants on the ascending giant branch.

6. Acknowledgements

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Table 1. Frequencies extracted by Period04

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Table 2. Modelling results: fundamental properties and $\chi^2$ for models with different masses. The expression of $\chi^2$ is given by Eq. (7), while $\chi^2_1$ takes frequencies of mixed modes with low mode inertia into consideration.

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<th>$M/M_\odot$</th>
<th>$Z$</th>
<th>Age (Gyr)</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>$L/L_\odot$</th>
<th>$R/R_\odot$</th>
<th>$\log g$</th>
<th>$\Delta \nu$ (\mu Hz)</th>
<th>$\chi^2$</th>
<th>$\chi^2_1$</th>
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Fig. 1.— Top panel: power density spectrum of the combined first five quarters of data (light-grey) and corresponding global model fit (black line). The dark-grey line is the smoothed (Gaussian with a FWHM of 0.5 µHz) power density spectrum. The dotted line is the fitted background and the dashed line is the white noise. The red line shows the contribution from activity, the blue from granulation and the green from faculaes (Karoff 2008). Bottom panel: background corrected power density spectrum in the range of the stellar oscillations. It is clear that $\nu_{\text{max}}$ is around 106 µHz and that peaks are regularly spaced with a large spacing of 9.37 µHz. Numbers are degrees of the modes. The multiple peaks corresponding to modes of $l = 1$ are believed to be mixed modes (Beck et al. 2011; Bedding et al. 2011).
Fig. 2.— The échelle diagram of the smoothed power density spectrum (dark line, FWHM of 0.1 \( \mu \)Hz) and the unsmoothed background corrected power density spectrum divided into bins each \( \Delta \nu \) wide. The red bars indicate 33 frequencies listed Table 1. The peaks for the same degree almost line up. The offset from perfect alignment is because the large frequency spacing changes with the frequency. Multiple peaks for \( l = 1 \) modes can be seen clearly.
Fig. 3.— Evolutionary tracks for eight models with different initial masses and heavy element abundances indicated by different colours listed in Table 2. The rectangle is the 1σ error box for the observation constraints, and dots are models which have a large frequency spacing close to 9.37 µHz.
Fig. 4.— Échelle diagrams for models plotted in Fig. 3. Squares, diamonds, triangles and circles are used for modes of degree $l = 0, 1, 2,$ and $3$, respectively. Observed frequencies are indicated by filled triangles. For theoretical frequencies, only those with a corresponding observed mode are shown (except for $l = 0$ modes) and symbol size indicates expected amplitude of each mode, which is scaled from mode inertia. For the observed ones, the size is scaled from the amplitude listed in Table 1. Models with masses around $1.43 \, M_\odot$ reproduce the observed frequencies better than those with higher masses. The differences between individual frequencies are included in the calculation of $\chi^2_1$. 
Fig. 5.— Mode mass for each degree (squares linked by solid line: \( l = 0 \), diamonds linked by dashed line: \( l = 1 \), triangles linked by dash dot line: \( l = 2 \), circles linked by dotted line: \( l = 3 \)) versus frequency for the model with mass of 1.43 \( M_\odot \) that has the smallest \( \chi^2 \). The mode inertia is the ratio of mode mass to stellar mass. Those frequencies with minimal mode masses (hence minimal mode inertias) are plotted in the échelle diagram in Fig. 4.