Calibrating Convective properties of Solar-like Stars in the *Kepler* Field of View

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ABSTRACT

Stellar models generally use simple parametrizations to treat convection. The most widely used parametrization is the so-called "Mixing Length Theory" where the convective eddy sizes are described using a single number, α, the mixing-length parameter. This is a free parameter, and the general practice is to calibrate α using the known properties of the Sun and apply that to all stars. Using data from NASA's *Kepler* mission we show that using the solar-calibrated α is not always appropriate, and that in many cases it would lead to estimates of initial helium abundances that are lower than the primordial helium abundance. *Kepler* data allow us to calibrate α for many other stars and we show that for the sample of stars we have studied, the mixing-length parameter is generally lower than the solar value. We studied the correlation between α and stellar properties, and we find that α increases with metallicity. We therefore conclude that results obtained by fitting stellar models or by using population-synthesis models constructed with solar values of α are likely to have large systematic errors. Our results also confirm theoretical expectations that the mixing-length parameter should vary with stellar properties.

Subject headings: stars: fundamental parameters — stars: interiors — stars: oscillations

1. Introduction

Accurately treating convective heat transport in stellar models is difficult. The structure and evolution of most stars is related to convective transport processes in their outer layers. The transition from efficient convective transport in the deep envelope to the radiative atmospheric layers takes place in a region of inefficient convection where the temperature gradient is highly superadiabatic. The poorly known structure of this region remains one of the major uncertainties in stellar models. In one-dimensional calculations, this region is usually modeled using the "mixing length theory," or MLT (Böhm-Vitense 1958). This prescription assumes that one can approximate the full range of turbulent eddy-sizes by a typical size, and that an eddy on average travels a distance determined by the eddy size before losing its identity. This distance, is known as the "mixing length" and is usually defined as αH_p, where α is known as the "mixing-length parameter," and H_p is the local pressure scale height. Given the atmospheric structure, the approximation, when given the mixing length, fix the specific entropy in the deep convection zone, which in turn determines the radius of the model. The model radius thus depends sensitively on the choice of the mixing-length parameter α, which is a free parameter that cannot be determined from the mixing length theory.

The common practice when modeling stars to determine their structure and evolution is to use the solar value of α. We know the mass, radius, luminosity and age of the Sun precisely. Solar models are constructed by searching for α and the initial helium abundance Y_0 that yield a model with the correct radius and luminosity at the Sun's age. Since masses, radii and ages are usually unknown for other stars, the solar approach is unfeasible and, hence, the solar value of α is used.

The assumption of a fixed α has no *a priori* justification, and indeed, there is some evidence that the solar value of α does not always work for other stars. Lattanzio (1984) first noted that the radius of α Cen A cannot be reproduced using the solar value of α. A combined astrometric and seismic study of the α Cen system by Demarque et al. (1986) confirmed this result as have more recent studies (e.g., Fernandes & Neuforge 1995; Miglio & Montalban 2005). Some studies of other binary systems have also suggested a mass-dependence of α (e.g., Ludwig & Salaris 1999; Morel et al. 2000; Lebreton et al. 2001; Lastennet et al. 2003, etc.). In particular, Yıldız et al. (2006) studied binaries in the Hyades cluster and suggested that α increases with stellar mass. Numerical simulations of stellar convection also suggest that convective properties vary with stellar parameters (e.g., Ludwig et al. 1999; Trampedach 2007; Trampedach & Stein 2011).

While studies of binaries and clusters have suggested that the solar α is not always applicable, the situation for single field stars is not clear because of the lack of observational constraints. Asteroseismic data from space missions like CoRoT (Michel et al. 2008) and *Kepler* (Borucki et al. 2010) allow us to place independent constraints on the mass and radius of single stars. These constraints, along with the classical constraints of $T_{\rm eff}$ and metallicity, allow us to constrain α. We already know that the oscillation spectra of some CoRoT and *Kepler* stars cannot be reproduced using the solar value of α (e.g., Metcalfe et al. 2010; Deheuvels & Michel 2011, Mathur et al. 2012). Detailed asteroseismic modeling of the solar analogs 16 Cyg A & B has also required the adoption of non-solar values of α (Metcalfe et al. 2012).

In this study we use asteroseismic data obtained by NASA's *Kepler* mission to calibrate α for a sample of dwarfs and subgiants. Observational data and our analysis technique are described in § 2. We present our results and discuss their implications in § 3.

It should be noted that α is just a proxy for describing stellar convection, and in particular α describes the entropy change between the surface and the deeper, isentropic, layers of efficient convection. As a result, the value of α cannot be derived uniquely. It depends on the exact formulation of the mixing-length theory (see, e.g., Appendix A of Ludwig et al. 1999), as well as physics inputs that affect entropy e.g., atmospheric opacities, the temperature-optical depth relation (T-τ) in the atmosphere, the equation of state and processes such as the gravitational settling of helium and heavy elements. Thus, the value of α in a given star needs to be examined in the context of the value of α needed to construct a solar model with the same physics.

Note that conventional MLT assumes a fixed ratio between the distance traveled by an eddy and its size, and α is the only free parameter. Some approximations (see e.g., Arnett et al. 2010) leave the ratio as an adjustable parameter. We use the conventional Böhm-Virtense form of MLT and only adjust α.

2. Method

We used data obtained by the *Kepler* asteroseismology program (Gilliland et al. 2010) during its survey phase. The survey collected data on more than 2000 main-sequence and sub-giant stars and stellar oscillations were detected in about 500 (Chaplin et al. 2011). Of these we used a subset of 90 stars for which spectroscopic estimates of effective temperature $T_{\rm eff}$ and metallicity [M/H] are available from Bruntt et al. (2012).

We used the average large separation $\Delta\nu$ and the frequency of maximum oscillation power $\nu_{\rm max}$ for this work. The large separation scales approximately as the square root of the mean density of a star (Ulrich 1986; Christensen-Dalsgaard 1988) while the frequency of maximum power scales approximately as $g/\sqrt{T_{\rm eff}}$ (Brown et al. 1991; Kjeldsen & Bedding 1995; Bedding & Kjeldsen 2003). The $\Delta\nu$ and $\nu_{\rm max}$ values are those used by Verner et al. (2011) to verify the *Kepler* Input Catalog (Brown et al. 2011).

First, $\Delta\nu$, $\nu_{\rm max}$, $T_{\rm eff}$ and [M/H] are used to estimate the mass and radius of each star. For this we used the grid-based Yale-Birmingham (YB) pipeline described by Basu et al. (2010) Gai et al. (2011) and Basu et al. (2012). We use four stellar-model grids for our work: models from the Yonsei-Yale (YY) isochrones (Demarque et al. 2004), and those of Dotter et al. (2008), Marigo et al. (2008) and Gai et al. (2010). The grids have been constructed with different physics inputs and modeling parameters, and with solar α consistent with the inputs to the grids. For subsequent calculations the average of the mass and radius estimates returned by the four grids is used.

The properties of our final sample are shown in Fig. 1. Note that our sample is quite restricted in terms of $\log g$ and we lack stars that are close to the base of the red giant branch;

in fact we have very few stars with $\log g < 4$. This will make finding a $\log g$ dependence of the derived α difficult. There is the expected correlation between mass and $T_{\rm eff}$ — the higher temperature stars are generally more massive. Although each mass range spans a range of $\log g$, unsurprisingly the least evolved stars in our sample are also the least massive. Most stars in our sample have sub-solar metallicities.

Starting with mass, radius, $T_{\rm eff}$ and metallicity, each star in our sample was modeled using the Yale Stellar Evolution Code (YREC; Demarque et al. 2008) in an iterative manner. In this mode, radius and $T_{\rm eff}$ were specified and the code determined either α or the initial helium abundance (Y_0) that yielded the specified radius at the given $T_{\rm eff}$ for a given mass. In the former case Y_0 has to be specified, in the latter α has to be specified.

The input physics consisted of the OPAL equation of state (Rogers & Nayfonov 2002), OPAL high-temperature opacities (Iglesias & Rogers 1996) supplemented with Ferguson et al. (2005) low temperature opacities. Nuclear reaction rates were from Adelberger et al. (1998), except for the $^{14}N(p,\gamma)^{15}O$ reaction, which was fixed at the value of Formicola et al. (2004). Models did not include core overshoot or the diffusion and settling of helium and heavy elements. We used the Eddington $T-\tau$ relation in the atmosphere. With the above physics, the solar-calibrated value of α is 1.690. Including gravitational settling of helium and metals would change that to 1.826.

For the first set of calculations we assumed solar α for all stars and determined Y_0 that would be needed to model the stars. We then performed two other sets of calculations: (1) we estimated α assuming that all stars have the solar value of $Y_0 = 0.278$ (Serenelli & Basu 2010); (2) we estimated α assuming that Y_0 follows a simple chemical evolution model, $Y_0 = 0.245 + 1.54Z$ (Dotter et al. 2008).

The iterative modeling process was repeated for 200 Monte Carlo realizations of $R,\ M,\ T_{\rm eff}$ and [M/H] to estimate the uncertainties in Y_0 (or α). Parameters in each realization were randomly chosen from a Gaussian distribution centered on the measured central value, with the dispersion equal to the mean measurement error. To avoid uncertainties due to small number statistics, we show results of stars with at least 20 converged iterations. This requirement resulted in a final sample of about 55 stars. The median of the distribution of parameter values is quoted as the central value of the parameter, and uncertainties are determined as the 68% confidence limit of the distribution.

The reliability of the derived α estimates depends on the reliability of the observations and those of the mass and radius estimates. There are indications that grid-based mass estimates can have systematic errors caused by differences in the input physics of the grids; however, these are smaller than those caused by uncertainties in $T_{\rm eff}$, [M/H], $\Delta\nu$ and $\nu_{\rm max}$

(Basu et al. 2012). Mathur et al. (2012) have shown that grid-based estimates of stellar masses and radii agree very well with those obtained from more detailed modeling of the oscillation frequencies, giving us confidence in the robustness of our mass and radius estimates. While detailed modeling of the oscillation spectrum is preferable for all stars, this is beyond the scope of this paper.

3. Results and Discussion

The initial helium abundance, Y_0, needed to construct models of the stars in our sample — assuming that they all have the solar value of α — is shown in Fig. 2. Note that for $> 50\%$ of our sample the Y_0 estimate is less than the primordial value of $Y_\mathrm{p} = 0.2477 \pm 0.0029$ (Peimbert et al. 2007). While the deficit is within 1σ for some stars, there are stars with a $> 3\sigma$ deficit. The weighted average of Y_0 of the sample is 0.227 ± 0.004. The weighted median is 0.223, both below Y_p. It is, of course, highly unlikely that stars are born with less helium than was produced in the Big Bang. Given that the only parameter we could change in MLT is α, this implies that solar α does not properly approximate convective heat transport in these stars. Note that a change of physics inputs to the models will change the solar value of α, and the exercise repeated with the new solar α would give similar results.

In Fig. 3(a) we show the value of α for our sample obtained assuming either the solar value of Y_0 (red points), or the simple chemical evolution model of Y_0 (black points). Note that α for most of the stars is less than the solar value for both cases The average value of α for this sample is 1.522 for solar Y_0 and 1.597 when the chemical-evolution model is used. In MLT, lower α implies less efficient convection. Thus MLT predicts that in the superadiabatic layers, convective energy transport in our sample is generally less efficient than that in the Sun. Since the results with the two choices of Y_0 are similar, in the subsequent discussions we only use α obtained with the chemical evolution model of Y_0. Mathur et al. (2012) constructed detailed models to fit the mode frequencies of 22 *Kepler* stars using the Asteroseismic Modeling Portal (AMP; Metcalfe et al. 2009) There are 16 stars in common with our sample. In Fig. 3(b) we show the differences between the Mathur et al. α values and the ones obtained in this work. The two α estimates agree well, mostly within 1σ. Since the physics in the AMP models is different from ours, they obtain a solar α of 2.12. Thus to compare their results with ours, we have scaled the AMP results to our value of the solar α. In Fig 3(c) and (d) we show the variation of α with $\log g$ and [M/H].

In order to explore whether our α estimates are correlated with stellar properties, we first determined the simple Spearman rank correlation between α and different properties. The correlation coefficients are listed in Table 1. Also listed is the p-value, which is the probability that the correlation is a chance occurrence. A small p therefore indicates a significant correlation. There thus appears to be significant correlation between metallicity and α. There also seems to be a mildly significant correlation between mass and α. The significance of the correlation of α with $\log g$ or $T_{\rm eff}$ depends on whether or not we include the low-$\log g$ stars in our analysis. Table 1 lists the coefficient obtained for the entire sample, as well as that obtained by removing the lowest $\log g$ stars ($\log g < 3.8$) in our sample.

Since α depends simultaneously on a number of parameters, to get a better estimate of the correlations we perform a trilinear fit to α with the model

$$\alpha = a + b \log g + c \log T_{\rm eff} + d[{\rm M/H}]. \tag{1}$$

Table 1 lists the coefficients and p-values, and Fig. 4 shows the residuals, and partial residuals, of the fit to Eq. 1. Note that the metallicity dependence is robust. The $\log g$ and $T_{\rm eff}$ correlations are small and less statistically significant when the entire sample is used; these increase in significance, but change signs, when the $\log g$ cut is applied. The mass correlation seen in the Spearman correlation is most likely to be the result of the mass-$T_{\rm eff}$ and mass-$\log g$ correlation seen in Fig. 1 as indicated by the fact that the residuals of the fit to Eq. 1 do not show any trend with mass (Fig. 4(a) and (e)).

The metallicity dependence of α is relatively easy to understand. It is most likely caused by the temperature sensitivity of the H^- density and hence, of its dominant contribution to the optical continuum opacity. E.g., in the solar photosphere, $\sim 50\%$ of the electrons that form H^- are donated by metals, and the fraction increases steeply with height due to low-ionization potential elements like Na, Al, K, Ca and Cr. With a smaller amount of metals, the temperature sensitivity of the H^- density will therefore increase. This in turn will increase the contrast between up- and down-flows, and especially increase the range of depths over which the down-flows will be cooled. This results in a larger entropy jump between the surface and the deeper layers, meaning a lower convective efficiency (a smaller α in the context of MLT). Since the average metallicity of our sample is sub-solar, we believe that this metallicity dependence accounts for the lower-than-solar average value of α for our sample.

The lack of a significant correlation between α and $T_{\rm eff}$ or $\log g$ is surprising. This is most likely the result of the limited and skewed range of $\log g$ of our sample, and is confirmed by the change of the sign of the correlation when the $\log g$ cutoff is applied. A larger sample should resolve these issues, in particular, data on giants should help determine the $\log g$ dependence properly. Piau et al. (2011), using a sample of red giants with radii known from interferometry, have shown that the red-giant models require sub-solar values of α to explain the observations; however, they did not address the dependence of α on stellar parameters.

As noted earlier, α is just a proxy for describing convection in stars. Although it is known that using such a proxy does not reproduce properties of the stellar near-surface layers correctly, MLT remains a practical tool in stellar modeling. An α type parameter can also be derived from numerical simulations of stellar convection (e.g., Ludwig et al. 1999; Trampedach et al. 1999; Trampedach 2007). At present it is difficult to compare these results with ours since the simulations were for solar composition, and the metallicity dependence of our results is fairly strong. However, there do seem to be some differences between our findings and the simulations. At a given $\log g$, Ludwig et al. (1999) found α to decrease with increasing $T_{\rm eff}$ for dwarfs in their 2D simulations and they find α to decrease with $\log g$. A similar behavior was seen in the 3D simulations of Trampedach (2007). Our α-$T_{\rm eff}$ correlation agrees with theirs, but the $\log g$ one does not when we examine the entire sample; when we apply the $\log g$ cutoff the reverse becomes true, the $\log g$ correlation agrees, the $T_{\rm eff}$ correlation does not.

As more detailed asteroseismic data become available from *Kepler*, and they are modeled, we will be able to reduce the uncertainties in the mixing-length parameters needed to model the stars, and the dependence of the properties of near-surface convection, including the corresponding value of α, on stellar parameters will become clearer. As it is, our results have important implications for the different branches of astrophysics that depend on fitting stellar models. The usual way to determine stellar properties is through spectroscopic or photometric analyses of the star combined with fitting to grids of stellar models to obtain masses and radii (e.g., Takeda et al. 2007) or ages (e.g., Jørgensen & Lindegren 2005). At a given metallicity, a lower α makes the evolutionary track of a given mass redder than its higher-α counterpart, and thus the properties of a star obtained using models constructed with a solar α will be quite different from those obtained with a sub-solar α. Although the $T_{\rm eff}$ change with α is really a result of a radius change, it appears as a change in the estimated mass of star being fitted (Basu et al. 2011). Consequently, we would be underestimating the mass of a sub-solar metallicity star if we use models constructed with the solar value of α. Stellar population and spectral synthesis models also use a single (usually the solar calibrated) value of α (see e.g. Coelho et al. 2007), and our results now show that the uncertainties in α need to be added to the error budget of results that use those models.

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Table 1: Correlations and the associated p value between α and stellar parameters.

	Spearman Correlation			
	All data		$\log g \geq 3.8$	
Parameter	r	p-value	r	p-value
$\log g$	0.110	0.436	0.316	0.027
$\log(T_{\text{eff}})$	0.039	0.781	0.224	0.122
[M/H]	0.445	0.001	0.636	10^{-5}
Mass	0.154	0.274	0.263	0.067
	Trilinear Analysis			
	All data		$\log g \geq 3.8$	
	fitted-value	p	fitted-value	p
a	7.97 ± 0.27	0.010	-12.77 ± 2.91	6.8×10^{-5}
b	-0.31 ± 0.09	0.002	0.54 ± 0.11	1.7×10^{-5}
c	-1.33 ± 0.80	0.102	3.18 ± 0.69	3.3×10^{-5}
d	0.48 ± 0.12	2×10^{-3}	0.52 ± 0.07	4.5×10^{-9}

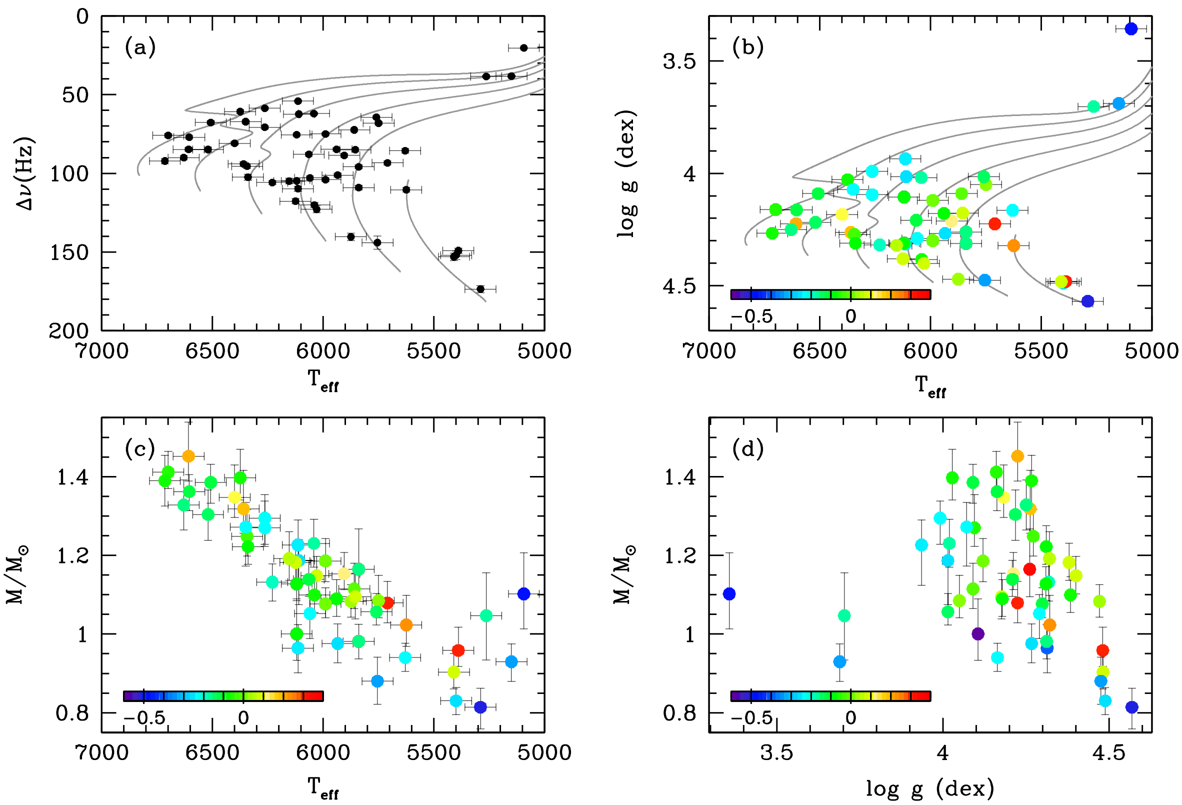

Fig. 1.— (a) $\Delta\nu$ of the stars plotted against $T_{\rm eff}$. The gray lines are tracks for masses 0.9 $M_\odot$ to 1.4 $M_\odot$; (b) Derived $\log g$ plotted against $T_{\rm eff}$; (c) The derived mass as a function of $T_{\rm eff}$; and (d) the derived mass plotted as a function of the derived $\log g$. In panels (b)-(d) the color scale denotes metallicity (dex).

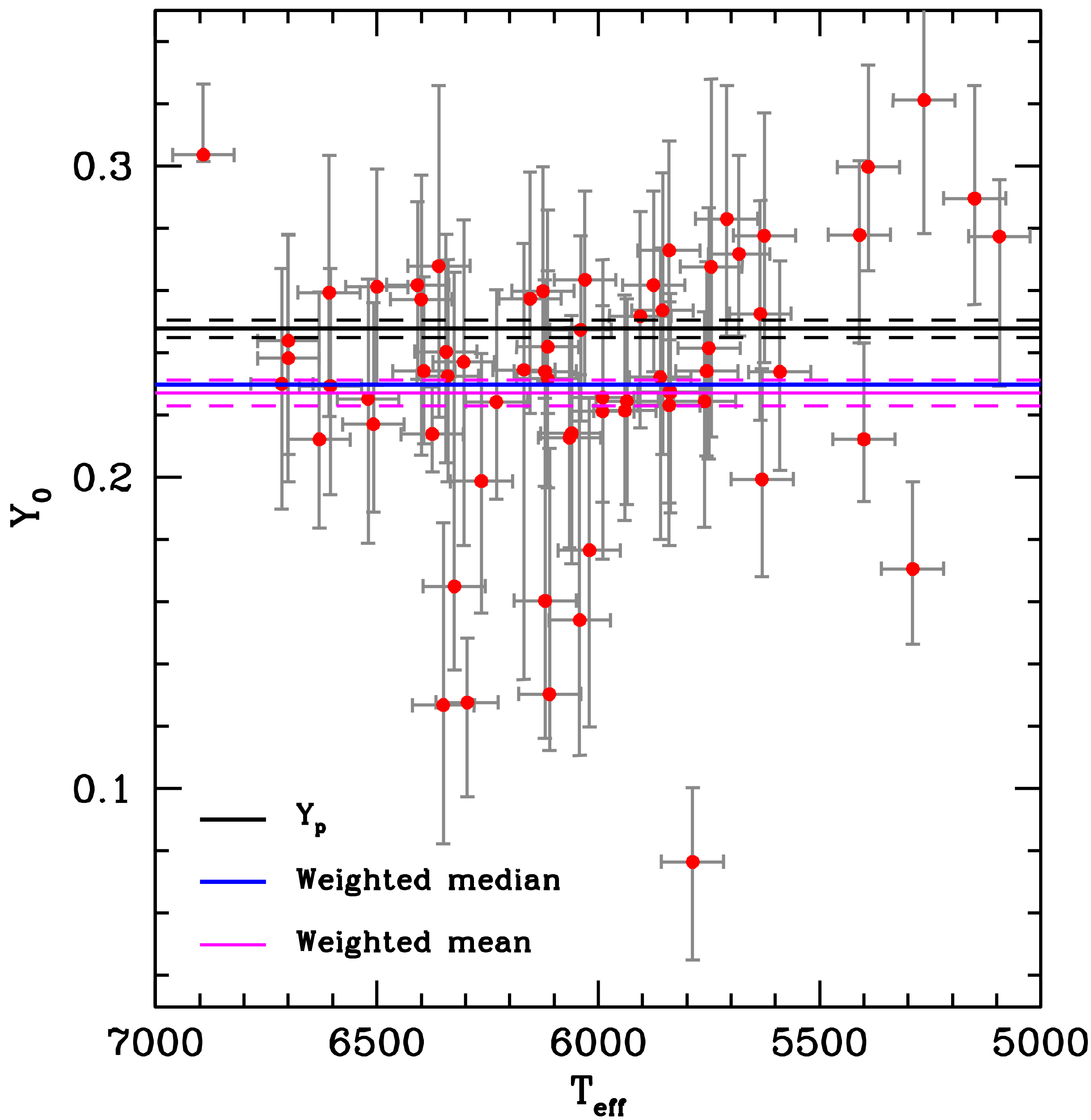

Fig. 2.— The initial helium abundance, Y_0, of stars in our sample obtained with the solar value of α. The black line marks the primordial Big Bang nucleosynthesis helium fraction, $Y_p = 0.2477 \pm 0.0029$. The pink line is the weighted mean of the sample, and the blue is the weighted median. The dashed lines show 1σ errors.

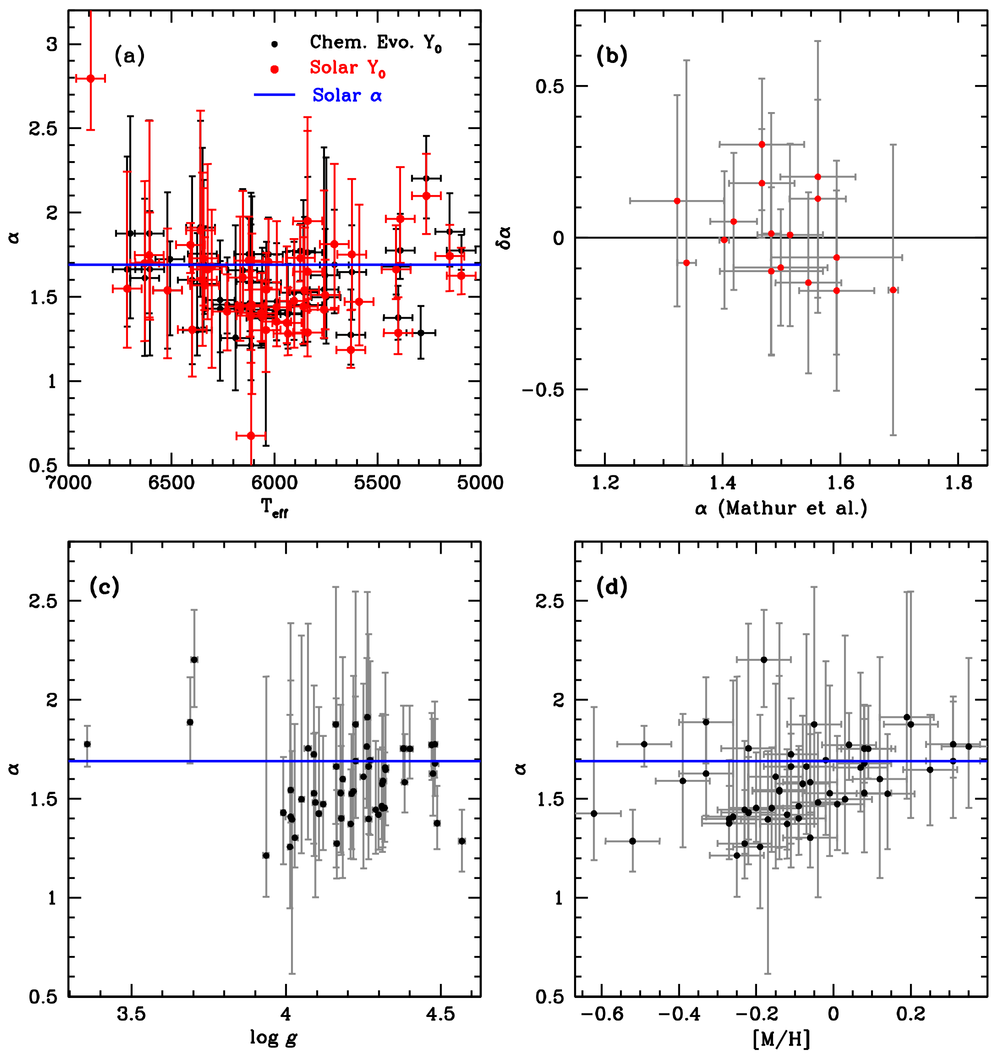

Fig. 3.— (a) The derived mixing length assuming the solar value of Y_0 (red) and for the chemical evolution model of T_0 (black). The blue line marks solar α. (b) The differences between α obtained by Mathur et al. (2012) by fitting individual frequencies and our α estimates of the same stars. (c) α plotted as a function of $\log g$, and (d) α plotted as a function of [M/H]. Only α estimated with the chemical evolution model of Y_0 are plotted in (c) and (d).

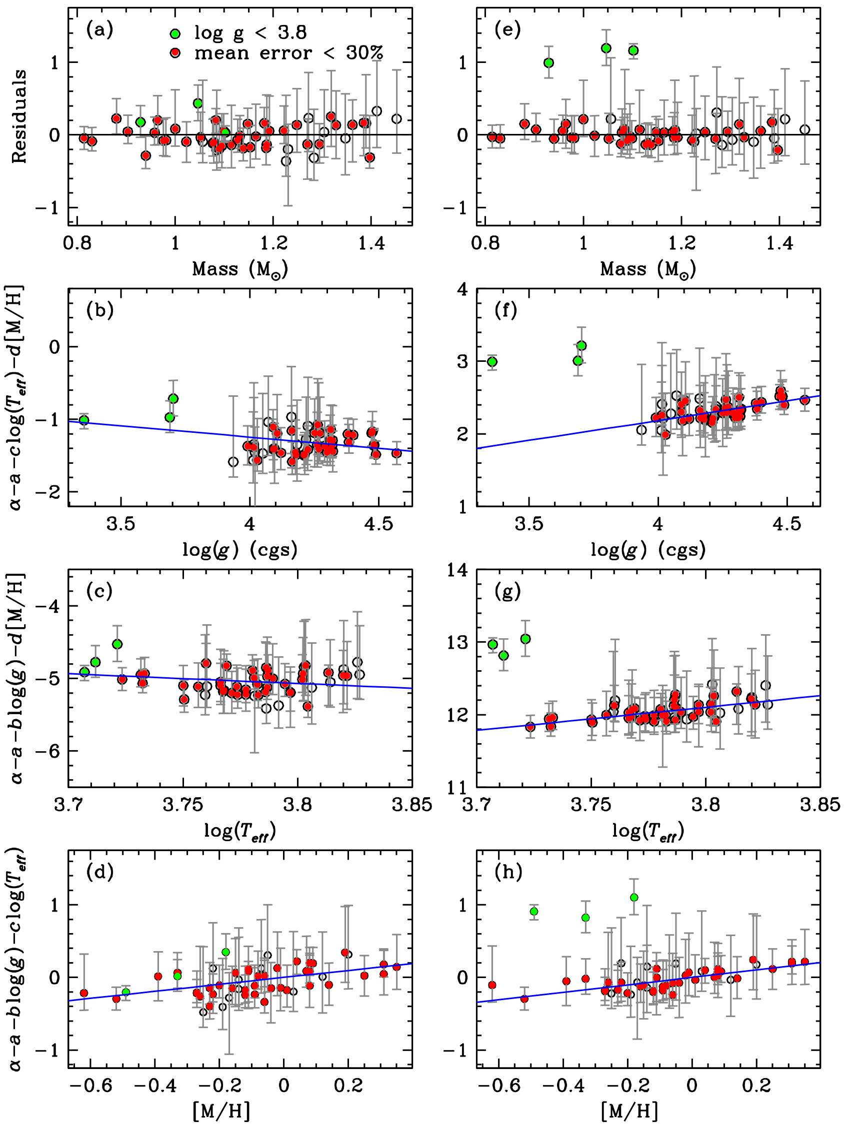

Fig. 4.— The residuals and partial residuals for the trilinear fit (Eq. 1). Panels (a)-(d) are results when the entire sample is used. Panels (e)-(h) are results for $\log g \geq 3.8$; the low $\log g$ points are however still shown. The intercept a changes when we apply the $\log g$ cutoff. The blue lines are $b \log g$ [panels (b), (f)], $c \log T_{\rm eff}$ [Panels (c),(g)] and d[M/H] [panels (d),(h)]. Note that the $\log g$ cut-off makes the correlations tighter and that the [M/H] relation remains almost unchanged with and without the $\log g$ cutoff.